BULLETIN

OF THE

AMERICAN MATHEMATICAL SOCIETY

EDITED BY

EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

HAROLD T. DAVIS HARRY W. KUHN

THEOPHIL H. HILDEBRANDT DERRICK N. LEHMER

JOHN R. KLINE HENRY L. RIETZ

CAROLINE E. SEELY

VOLUME XXXVIII, NUMBER 9, PART I

SEPTEMBER, 1932

PUBLISHED BY THE SOCIETY

MENASHA, WIS., AND NEW YORK

1932

PUBLISHED MONTHLY

Whole No. 396 $9.00 a Year

Entered as second class matter April 24, 1926 at the post office at Menasha, Wis., under the act of August 24, 1912.

This number is published in two parts, of which Part II is a list of members of this Society

AMERICAN MATHEMATICAL SOCIETY

Mail Address: 501 West 116th Street, New York, N.Y., for the Secretary, Treasurer, Librarian, and Clerk.

Office: Room 413, Physics Building, Columbia University, New York City.

President: Dean L. P. Eisenhart, Princeton University, Princeton, N.J.

Secretary: Dean R. G. D. Richardson, Brown University, Providence, R.I.

Associate Secretaries: Professor Tomlinson Fort, Lehigh University, Bethlehem, Pa.; Professor M. H. Ingraham, University of Wisconsin, Madison, Wis.; Dean T. M. Putnam, University of California, Berkeley, Calif.

Treasurer: Professor G. W. Mullins, Barnard College in Columbia University, New York City.

Librarian: Professor R. C. Archibald, Brown University, Providence, R.I.

Clerk: Dr. Caroline E. Seely, 413 Physics Building, Columbia University, New York City.

Editor-in-Chief of the Bulletin: Professor E. R. Hedrick, University of California at Los Angeles, Los Angeles, Calif.

Managing Editor of the Transactions: Professor R. D. Carmichael, University of Illinois, Urbana, Ill.

Chairman, Colloquium Editorial Committee: Professor R. L. Moore, University of Texas, Austin, Tex.

Managing Editor of the American Journal of Mathematics: Professor Abraham Cohen, Johns Hopkins University, Baltimore, Md.

Representatives of the Society in the Division of Physical Sciences of the National Research Council: Professors James Pierpont, D. R. Curtiss, and E. R. Hedrick.

Representatives on the Council of the American Association for the Advancement of Science: Professors Louis Ingold and M. H. Ingraham.

Society Visiting Lecturer for 1931-32: Professor R. L. Moore, University of Texas, Austin, Tex.

Colloquium Speaker for 1932: Professor J. F. Ritt, Columbia University, New York City.

Chairman, Committee on Arrangements for the Semi-Centennial: Professor T. S. Fiske, Columbia University, New York City.

THE SYRACUSE MEETING OF SECTION A AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

Section A (Mathematics) of the American Association for the Advancement of Science held sessions in the morning and afternoon of Tuesday, June 21st, 1932, at Syracuse, New York. About forty of those attending lunched together between the sessions at the University Cafeteria, drove to Green Lake in the late afternoon, and attended a picnic dinner.

At a joint session of Sections A and Q on Wednesday afternoon, Professor H. T. Davis of the University of Indiana presiding, Professor E. R. Hedrick of the University of California at Los Angeles showed how certain factors in the improvement of the teaching of mathematics depend on the proper cooperation between educationalists and mathematicians.

Among the other features of special interest to mathematicians were the report on the collegiate mathematics needed in the social sciences, presented at the Wednesday morning joint session of Section K and the Econometric Society, and the paper by Professor W. B. Carver of Cornell University on *Mathematics and the advancement of science*, given at the general session on Wednesday afternoon.

At the Tuesday morning session of Section A, Professor E. R. Hedrick, the retiring vice-president, presiding, papers were given by Professor H. M. Gehman of the University of Buffalo on *Homeomorphic geometry of the projective plane*, and by Professor W. A. Hurwitz of Cornell University on *Logical foundations for groups and fields*. At the afternoon session, Professor Virgil Snyder presiding, Professor J. A. Shohat of the University of Pennsylvania presented a paper on *Interpolation*.

Professor Gehman showed that analysis situs could be thought of as a branch of geometry in which the group of transformations of space is the group of homeomorphisms. The invariant properties of the following spaces were investigated: euclidean n-space, the euclidean plane, the inversion plane, the sphere, the projective plane. It was shown that the group of homeomorphisms of the projective plane which leave invariant the line at infinity form only a proper subgroup of the group of homeomorphisms of the euclidean plane, thus reversing the

situation that exists under the group of projective transformations. Finally he stated a theorem which gives the distinguishing characteristics of a transformation of this subgroup.

Professor Hurwitz surveyed various forms of definition of groups, abelian groups, and fields, by sets of postulates. Sets of as few as three postulates for groups have been given; in some cases the undefined operation representing the direct combination ab of elements of the group, in others the inverse combination ab^{-1}. For abelian groups, sets of as few as two postulates can be given, and for fields as few as four.

Professor Shohat dealt with various polynominal and trigonometric interpolation formulas, in the real and complex domains, emphasizing the two important properties of *convergence* (the function $f(x)$ to be interpolated assumed to be continuous) and *stability*, that is, the extent to which errors involved in the given values of $f(x)$ affect the interpolating function. (The degree of convergence was not discussed.)

For Lagrange's interpolating polynominal ($\equiv L.\ I.\ P.$) in the real domain, say $-1 \leqq X \leqq 1$, he showed that (1) it is impossible to choose an infinite system of nodes in $(-1, 1)$ so that the corresponding *L. I. P.* will be convergent for any continuous $f(x)$ over the whole interval, the best that can be obtained being convergence in case $f(x)$ satisfies a Dini-Lipschitz condition; and (2) the Lagrange interpolation formula is not stable. Dunham Jackson's trigonometric interpolation formula and Fejér's interpolating polynomial—a modification of *L. I. P.*—were both found to be stable and convergent for any continuous $f(x)$.

In the complex domain he discussed various *L. I. P.*, the nodes being suitably chosen—coinciding with the zeros of either orthogonal Tchebycheff polynominals, or Faber-Tchebycheff polynomials of best approximation, or, in the case of circular regions, coinciding with vertices of regular inscribed polygons. In all these cases convergence holds for an analytic $f(x)$.

Returning to the real domain he discussed a polynomial formula due to S. Bernstein which, although not an interpolation formula, employs a certain set of values of $f(x)$, has a very simple structure, and enjoys the two important properties—convergence for any continuous $f(x)$ and stability.

F. F. DECKER, *Acting Secretary*

JOHN WESLEY YOUNG—IN MEMORIAM

John Wesley Young* was born in Columbus, Ohio, November 17, 1879, and died in Hanover, New Hampshire, February 17, 1932, at the age of fifty-two years and three months.

It is fitting at this time to gather together a summary of the principal activities to which Professor Young gave of his energy, but the primary purpose of this article is to pay a sincere, but confessedly inadequate, tribute to Professor Young as a man and as a friend—as a man of many-sided ability with a long record of successes as research worker, teacher, administrator, committeeman, author, editor, and citizen; as a friend whose distinctive individuality and endearing personality remain real and living, even though their charm and vigor are now matters of memory alone.

John Wesley Young was the son of William Henry Young and Marie Louise Widenhorn Young. William Henry Young, born in West Virginia of native American parentage, had a varied and interesting career. After serving in the Civil War as a colonel, he was appointed by President Grant to the consular service in Germany. For a time he held a professorship at Ohio University in Athens, Ohio, but later retired to devote himself to business interests. Marie Louise Widenhorn was born in Paris, France, of a German father and a French mother. Previous to her marriage she had lived on the Continent and spoke both German and French with fluency.

When only a few years old, John Wesley took the first of many trips to Europe when his parents, ideally fitted for the task, took charge of a party of young people seeking a year of European travel and culture. Later, when his father's business interests required extended stays in South America, his mother maintained the family home in Germany. Hence, after four years of school divided between Karlsruhe, Germany, and Columbus, Ohio, John Wesley spent the six years from 1889 to 1895 in the Gymnasium at Baden-Baden. He then returned to the United States to complete his education.

Thus Young was not only born of an international marriage but also had the advantage of an international education,—an education by no means confined to formal learning from books, but one which included many lessons in the art of sane living in a world peopled with races and individuals of widely diversified standards, ideals, ambitions, and temperaments. Is it not reasonable to find in his early development and experiences a partial explanation of the unusual degree to which the mature man was open-minded on all questions, and uniformly tolerant of the views of others?

Although Young's years in German schools had many beneficial influences on his later life, Germany can claim little credit for his proficiency in mathematics. When he entered Ohio State University from the Gymnasium, he was granted advanced credits in both French and German, but was required

---

* A notice regarding Professor Young has appeared in the American Mathematical Monthly, vol. 39 (1932), pp. 309–314.

to take courses in mathematics in the associated preparatory department to supply his deficiency in entrance credits in that subject.

Young received the degree of Ph.B. at Ohio State University in 1899 and remained in residence for another year as the recipient of a graduate fellowship. He had specialized in both mathematics and philosophy without, as yet, giving either the preference. The balance was swung in favor of mathematics, partly through enthusiasm engendered by contacts with his talented brother-in-law, E. H. Moore, partly because Cornell University offered him a graduate scholarship for the study of mathematics.

At Cornell from 1900 to 1903, Young concentrated on mathematics and mathematical physics. In 1901 he was awarded the degree of A.M. and was elected to Sigma Xi. His thesis for the Master's degree, written under G. A. Miller, was entitled *On the holomorphisms of a group*, and was published without delay in the Transactions of this Society.*

For the next two years (1901–1903) Young worked for his doctorate under J. I. Hutchinson, Miller having gone to Stanford University. During this period he concentrated on automorphic functions. His thesis, *On the group of the sign* $(0, 3; 2, 4, \infty)$ *and the functions belonging to it*, appeared in the Transactions of this Society.† From February to June, 1903, he was seriously ill with typhoid.

The eight years from 1903 to 1911 found Young rapidly advancing from an instructorship at Northwestern to a full professorship at Kansas. In 1905 he went to Princeton, as preceptor, where he enjoyed fruitful association with Oswald Veblen, of which more will be said later. On July 20, 1907, he was married to Mary Louise Aston of Columbus, Ohio, a former schoolmate, who survives him. Their only child, Mary Elizabeth, is now Mrs. H. W. Allyn.

After two years as assistant professor at the University of Illinois, Young became the head of the department of mathematics at the University of Kansas, where he remained for one year. During the summer of 1911, he taught in the Summer Quarter of the University of Chicago.

These eight years were vitally significant for three reasons. They were the years in which Young devoted himself most assiduously to research. They were the years in which he gained a broad and intimate acquaintance with educational institutions of many varying types—coeducational and purely masculine, privately endowed and state controlled, collegiate and university, virtually every type except colleges exclusively for women. They were the years during which he formed many lasting friendships with fellow mathematicians whose roll embraces many of the present leaders. The value of this wide experience was evident in the skill with which he adapted himself to new conditions and the wisdom with which he solved new problems as they arose.

The remaining twenty-one years of his life, except for several leaves of absence, Young spent at Dartmouth College. Appointed originally as Professor of Mathematics on the Chandler Foundation, he was made B. P. Cheney Professor of Mathematics in 1912. He served as head of the department from

* Number 7 of the List of Professor Young's Publications at the end of this article. Hereafter, references to this list will be by number only.

† No. 10.

1911 to 1919, when the chairmanship system was adopted, and was appointed as chairman for the period 1923–1925.

For several reasons, it is difficult to state with accuracy the full extent of the individual credit due to Young for the progress made under his leadership. He was seldom dictatorial, and there was true democracy within the department. Important decisions were matters of group action and, while he freely gave credit to others for their good suggestions, he modestly disclaimed credit for many programs actively sponsored by others but indebted to his genius for their inception.

A comparison of recent Dartmouth catalogues with issues prior to 1911 reveals clear evidences of Young's influence on the courses offered. His interest in modern geometry, projective geometry (especially from the axiomatic point of view), and the fundamental concepts of mathematics, provided the inspiration for courses emphasizing these topics. Many of these courses were not intended solely for the specialist in mathematics, but were designed to afford the intellectually curious an opportunity, in a setting largely freed from technical difficulties, to gain an insight into the viewpoints of advanced mathematics as well as a keener appreciation of the logical aspects of elementary mathematics, and an understanding of the place of mathematics in the advance of civilization.

Young's influence was not restricted to courses in mathematics. He was instrumental in the establishment of the orientation courses in evolution and citizenship required of all freshmen at Dartmouth. He also organized a group of courses in the growth of science and devoted a great deal of time to studying and teaching the history of the development of the mathematical sciences.

Young was an ideal research worker—patient, philosophical, and thorough, but also highly imaginative, daring in the unconventionality of his attack, and extremely skillful in his clever handling of the technical difficulties so prevalent in worthwhile research. Moreover, he had the happy faculty of being able to present the fruits of his labors in a clear-cut fashion intelligible to others. At the end of this article is given a list of Young's papers. Comment here will be limited to a brief indication of the fields in which he worked.

His Master's thesis and a second paper* written during his year of work under Miller dealt with problems in abstract group theory. In his doctor's thesis,† written under Hutchinson, he studied in detail the monodromic group for the Riemann surface

$$y^4 = (x - k_1)^2(x - k_2)^2(x - k_3)(x - k_4)^3$$

of genus two, using methods developed by Klein in treating the elliptic modular group. He was thus led to a study of the associated θ-functions, which are a simple generalization of Poincaré's θ-Fuchsian functions, and particularly to the problem of the expansion in series of three fundamental functions in terms of which every θ-function of the group can be rationally expressed.

In his next paper‡ Young proposed a new line of attack, using hypercom-

* Nos. 7, 8.
† No. 10.
‡ No. 11.

plex numbers, upon the old unsolved problem of the arithmetical definition of the subgroups of the modular group. Then followed the first of three papers* on the discontinuous ζ-groups defined by normal curves of order n in space of n dimensions.

A paper† published in 1907 generalizes the Tchebychef problem of approximating to continuous functions by polynomials. Young replaces the polynomials by an arbitrary class of functions dependent on a finite number of parameters.

In 1908 Young shared with Veblen the distinction of creating a set of postulates for projective geometry‡ which won the approbation of the mathematical world by their simplicity and effectiveness. The work of Hilbert and others had made it clear that sound logical foundations could be built for the ordinary euclidean geometry but only at the expense of bringing into consideration concepts and methods, which, if introduced into a purely deductive treatment of geometry, transformed a supposedly elementary subject into one of formidable complexity.

Veblen and Young pointed out the advantages of first developing a "general projective geometry" from a simple set of assumptions, a procedure which permits of the postponement of the difficult topics of linear order and continuity, and hence greatly simplifies the logical abstract treatment of a considerable body of geometry. In 1910 Veblen and Young published a text in projective geometry§ in which this plan was carried out with thoroughly satisfactory results.

A series of papers‖ published between 1909 and 1916 shows a continued interest in groups, but chiefly in their relation to associated geometrical considerations. Thus projective geometry on a complex line is linked with the geometry of inversion in a real plane, and, as a later generalization (with F. M. Morgan as co-author), well known Cremona transformations of the third and higher degrees are interpreted by representing them as linear transformations of a hypercomplex variable.

As early as 1911¶ Young showed interest in the formulation of generalized conceptions in algebra, which first found expression in formal papers** in 1927.

During the last twenty years of his life administrative tasks made heavy demands upon his time and formal published papers were few in number, but his interest in research remained unabated. Even when spending many weeks in a hospital bed, shortly before his final illness, his mind was actively occupied with an attempt to construct an angle calculus designed to facilitate the study

---

* Nos. 12, 14, 18.

† No. 13.

‡ No. 15.

§ No. 1.

‖ Nos. 16, 17, 20.

¶ *On algebras defined by groups of transformations.* Read December 28, 1911. Abstract, this Bulletin, vol. 18 (1912), p. 216.

** Nos. 21, 22.

of those geometrical properties which are invariant under a similarity transformation.

An outstanding feature of Young's character was his really serious philosophical interest in any worthwhile problem, and his habit of keeping an absolutely open mind until full discussion brought out a convincing argument as to the side on which lay the truth.

He was never a conservative, but neither could he be fairly classed as a radical. He was that happy compromise suggested by the word progressive, that is, he was an ardent supporter of new ideas and methods but he never became so fanatical about them as to lose his sense of true values.

These traits made Young such an excellent administrative officer and committee member that many organizations availed themselves of his services. He became a member of the American Mathematical Society in February, 1902, was made an editor of the Bulletin and a member of the Council in 1907 and served steadily in these capacities for eighteen years. He served as a Vice-President from 1928 to 1930.

He aided in the creation of the Mathematical Association of America, became one of its Vice-Presidents in 1918, and its President from 1929 to 1931. The new organization appointed a committee on mathematical requirements with Young as chairman. In response to a widespread desire expressed both by secondary schools and by colleges that the definitions of mathematical requirements be revised, this committee was enlarged to make it nationally representative and was fully sponsored by the Mathematical Association of America with the generous financial assistance of the General Education Board. He served as the chairman of this National Committee from 1916 to 1923. From 1919 to 1921 Young was granted leave of absence to devote his whole time to this work. The final report* of the committee contained authoritative conclusions which have had and continue to have a far-reaching influence on mathematical instruction.

At the summer meeting of the Society in Providence in 1930 Young was appointed on an advisory committee to cooperate with the Society for the Promotion of Engineering Education which planned for the following summer a two-weeks summer school for teachers of mathematics to engineering students. It was at this summer school that his last paper,† *The adjustment between secondary school and college work in mathematics*, was given.

In 1918 Young was engaged in war work with the Educational Bureau of the Young Men's Christian Association with headquarters in New York City. He also spent some time in Washington, D. C., as a member of the Committee on Education and Special Training of the War Department.

For seven years, for four years as chairman, Young was a member of the Committee on Instruction at Dartmouth, and he also served on many other

---

* *The Reorganization of Mathematics in Secondary Education*; report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, Inc., 1923. x+652 pp.

† No. 29.

important committees of the college. From 1915 to 1917 he was Chief Examiner in Geometry for the College Entrance Examination Board.

His memberships in learned societies included most of the well known mathematical organizations of America and Europe, and he was a regular attendant at the International Congresses. He was also an active member of many other organizations such as the Dartmouth Scientific Association, Gamma Alpha, the New Hampshire Academy of Science, and the Connecticut Valley Section of the Association of Teachers of Mathematics in New England (of which he was President in 1926–1927).

He found time to write or edit a number of texts in mathematics. Mention has already been made of the *Projective Geometry* written with Oswald Veblen and published in 1910. His *Lectures on Fundamental Concepts of Algebra and Geometry*,* published in 1911, aroused widespread interest and was translated into Italian. In 1918 and 1919 he wrote with F. M. Morgan freshman texts in *Elementary Mathematical Analysis*† and *Plane Trigonometry*.‡ From 1924 until his death he was the mathematical editor for the Houghton-Mifflin Company and nearly a score of successful texts were published under his supervision.

Young also served in an editorial capacity for the Mathematics Teacher, the Colloquium Publications of the American Mathematical Society, and the Carus Mathematical Monographs. Number Four of the latter, *Projective Geometry*,§ was written by Professor Young. He also contributed an article on geometry‖ to the fourteenth edition of the Encyclopedia Britannica.

Such is the partial record of the busy life led by John Wesley Young. For several decades he has been an inspiring leader in the mathematical life of this country, while his sincere friends may be found in many parts of the world.

He carried on to the very end of his life. After spending most of the first semester of the present academic year in the hospital, he recovered sufficiently to take up his college work in the second semester. His strength was equal only to a restricted teaching schedule, but he met his classes and taught them with his usual effectiveness to the end. When he left his class at noon of February 15, he was taken directly to the hospital, to await courageously the final summons, which came before the class had met again.

In the words of part of the faculty resolution, "It is true that the Faculty of Dartmouth College has lost one of its most distinguished members. But in a more personal way we shall miss his calm, sometimes whimsical, and always logical and unprejudiced treatment of matters under discussion in meetings of the faculty. We shall miss his smiling greetings on the street and his kindly interest in our doings. We shall miss those stimulating conversations that we have had with him on matters far from mathematical."

---

* No. 2.
† No. 4.
‡ No. 5.
§ No 6.
‖ No. 27.

LIST OF PUBLICATIONS

Books

1. *Projective Geometry* (with Oswald Veblen), vol. 1, Ginn & Co., 1910. x+342 pp. (Vol. 2, published under authorship of Veblen and Young, was written by Veblen alone.)
2. *Lectures on Fundamental Concepts of Algebra and Geometry*, Prepared for publication with the coöperation of William Wells Denton with a note on the Growth of Algebraic Symbolism by U. G. Mitchell. The Macmillan Company, 1911. vii+247 pp.
   Italian Translation by D. Mercogliano: *I Concetti Fondamentali dell' Algebra e della Geometria*. Naples, L. Pierro, 1919.
3. *Plane Geometry* (with A. J. Schwartz), Henry Holt & Co., 1915. x+223 pp. Second edition, 1923.
4. *Elementary Mathematical Analysis* (with F. M. Morgan), The Macmillan Company, 1918. xii+548 pp.
5. *Plane Trigonometry and Numerical Computation* (with F. M. Morgan), The Macmillan Company, 1919. vii+122 pp.
6. *Projective Geometry*. The Carus Mathematical Monographs, No. 4. Open Court Publishing Co., 1930. ix+185 pp.

Articles on Mathematical Research

7. *On the holomorphisms of a group*, Transactions of this Society, vol. 3 (1902), pp. 186–191.
8. *On a certain group of isomorphisms*, American Journal of Mathematics, vol. 25 (1903), pp. 206–212.
9. *A simple existence proof for logarithms*, American Mathematical Monthly, vol. 10 (1903), pp. 227–230.
10. *On the group of the sign* (0, 3; 2, 4, ∞) *and the functions belonging to it*, Transactions of this Society, vol. 5 (1904), pp. 81–104.
11. *The use of hypercomplex numbers in certain problems of the modular group*, this Bulletin, vol. 11 (1905), pp. 363–367.
12. *A class of discontinuous ζ-groups defined by normal curves of the fourth order in a space of four dimensions*, Rendiconti del Circolo Matematico di Palermo, vol. 23 (1907), pp. 97–106.
13. *General theory of approximation by functions involving a given number of arbitrary parameters*, Transactions of this Society, vol. 8 (1907), pp. 331–344.
14. *A fundamental invariant of the discontinuous ζ-groups defined by the normal curves of order n in space of n dimensions*, this Bulletin, vol. 14 (1908), pp. 363–367.
15. *A set of assumptions for projective geometry* (with O. Veblen), American Journal of Mathematics, vol. 30 (1908), pp. 347–380.
16. *The geometry of chains on a complex line*, Annals of Mathematics, (2), vol. 11 (1909–10), pp. 33–48.

17. *Two-dimensional chains and the associated collineations in a complex plane*, Transactions of this Society, vol. 11 (1910), pp. 280–293.
18. *The discontinuous ζ-groups defined by rational normal curves in a space of n dimensions*, this Bulletin, vol. 16 (1910), pp. 363–368.
19. *Fundamental regions for cyclical groups of linear fractional transformations on two complex variables*, this Bulletin, vol. 17 (1911), pp. 340–344.
20. *The geometries associated with a certain system of Cremona groups* (with F. M. Morgan), Transactions of this Society, vol. 17 (1916), pp. 233–244.
21. *On the partitions of a group and the resulting classification*, this Bulletin, vol. 33 (1927), pp. 453–461.
22. *A new formulation for general algebra*, Annals of Mathematics, (2), vol. 29 (1927), pp. 47–60.

MISCELLANEOUS

23. *On some tendencies in geometric investigations*; "Remarks addressed to my students" by Corrado Segre, translated by J. W. Young, this Bulletin, vol. 10 (1904), pp. 442–468.
24. *The present and the future of mathematical physics*; address delivered before the section of applied mathematics of the International Congress of Arts and Science, St. Louis, September 24, 1904, by Henri Poincaré; translated by J. W. Young, this Bulletin, vol. 12 (1906), pp. 240–260.
25. *Remarks to problem* 409, American Mathematical Monthly, vol. 22 (1915), p. 270.
26. *The organization of college courses in mathematics for Freshmen*; read at the meeting of the Mathematical Association of America at the University of Rochester, September 5, 1922, American Mathematical Monthly, vol. 30 (1923), p. 6.
27. *Geometry*, Encylopedia Britannica, fourteenth edition, 1929.
28. *Functions of the Mathematical Association of America*; retiring presidential address delivered at Minneapolis, September 8, 1930, American Mathematical Monthly, vol. 39 (1932), pp. 6–15.
29. *The adjustment between secondary school and college work in mathematics*, Journal of Engineering Education, vol. 22 (1932), pp. 586–595.

Reviews in this Bulletin: vol. 12, pp. 127, 128, 399; vol. 13, p. 405; vol. 14, p. 147; vol. 15, p. 463; vol. 16, p. 254; vol. 17, pp. 42, 258; vol. 18, p. 193; vol. 19, p. 370; vol. 27, p. 484; vol. 28, pp. 224, 416, 469, 471; vol. 29, pp. 38, 40, 185, 232, 271, 470, 480; vol. 30, pp. 88, 182, 278; vol. 34, p. 788.

R. D. BEETLE
C. E. WILDER

RAMSEY ON FOUNDATIONS OF MATHEMATICS

The Foundations of Mathematics, and other Logical Essays. By Frank Plumpton Ramsey. Edited by R. B. Braithwaite. Preface by G. E. Moore. (International Library of Psychology, Philosophy, and Scientific Method.) New York, Harcourt, Brace and Company; London, Kegan Paul, Trench, Trubner and Company, Ltd., 1931. xviii+292 pp. $4.50.

Ramsey's *The Foundations of Mathematics* is a collection of eighteen papers all bearing in some way on logic. The papers range in date from 1923 to 1929, and have been edited after the death of the author in January, 1930, at age 27. Besides these writings and the Preface, the book also contains the editor's Introduction, his Bibliography of Ramsey, and his Note on Symbolism. The author was Lecturer in Mathematics in the University of Cambridge; the editor is Fellow of King's College, Cambridge; and Professor Moore, who wrote the preface, is Professor of Mental Philosophy and Logic at Cambridge.

Of Ramsey's eighteen papers, the first sixteen are divided into nine groups, I–IX; the remaining two are in the form of an Appendix and an Epilogue. Some idea of the nature of the papers in groups I–IX can be had from a mere inspection of their titles: (I) The Foundations of Mathematics; (II) Mathematical Logic; (III) On a Problem of Formal Logic; (IV) Universals; (V) Note on the Preceding Paper; (VI) Facts and Propositions; (VII) Truth and Probability; (VIII) Further Considerations—consisting of (A) Reasonable Degree of Belief, (B) Statistics, (C) Chance; (IX) Last Papers—consisting of (A) Theories, (B) General Propositions and Causality, (C) Probability and Partial Belief, (D) Knowledge, (E) Causal Qualities, (F) Philosophy.

The Appendix is a reprint of a critical review of Wittgenstein's *Tractatus Logico-Philosophicus.* The Epilogue touches lightly on a variety of subjects.

The papers are very uneven as to length and as to clearness and literary finish. Paper (B) of group (VIII), for instance, is scarcely more than a page long, and consists merely of a few rather disconnected remarks on statistics. Paper (I), on the other hand, is 61 pages in length, and is an admirably organized discussion of the logic of *Principia Mathematica* as a basis for mathematics. This unevenness in the length and in the literary quality of the papers is due to the fact that the editor has included in the book papers previously published, as well as papers never before published. It is among the hitherto unpublished writings that one finds the very short and somewhat obscure papers. Some of these are mere notes which were not intended by the author to be published in their present form, but were meant to be developed into publishable forms later.

To the mathematical reader, the papers that appear to be of special interest are (I), (II), (III), (VII), and (A) of (IX). In (I), whose title forms the principal title of the book, the author aims "to give a satisfactory account of the Foundations of Mathematics in accordance with the general method of Frege, Whitehead, and Russell. Following these authorities, I hold that mathematics is part of logic . . . I have therefore taken *Principia Mathematica* as a basis for

discussion and amendment; and believe myself to have discovered how, by using the work of Mr. Ludwig Wittgenstein, it can be rendered free from the serious objections which have caused its rejection by the majority of German authorities." The parts of the *Principia* which thus receive "discussion and amendment" concern the definition of *class*, the Axiom of Reducibility, the definition of *identity*, the Multiplicative Axiom, and the Axiom of Infinity.

In paper (II), the author discusses in outline the logical work of Weyl, Brouwer, and Hilbert, which has appeared since the publication of the *Principia* and which "claims to supersede altogether the position taken up by Whitehead and Russell as to the nature of mathematics and its logical foundations."

In (III), the author is concerned with the solution of a special case of Hilbert's *Entscheidungsproblem* in mathematical logic.

In (VII), there is developed "a purely psychological method of measuring belief," and "the theory of probability is taken as a branch of logic, the logic of partial belief and inconclusive argument."

In (A) of (IX), a mathematical theory is regarded "simply as a language for discussing the facts the theory is said to explain."

To the worker in Philosophy, the papers that should prove most interesting are those that have not hitherto been published—those in groups VII–IX. These are, in the main, the rough notes of a fertile mind on fundamental problems in philosophy.

All readers will feel, throughout the book, that they are in the presence of a penetrating intellect, a clear and effective writer, and a charming personality. It is true that Ramsey himself regards his work in connection with the *Principia*—probably his most important work—only as an essay: "although my attempted reconstruction of the view of Whitehead and Russell overcomes, I think, many of the difficulties, it is impossible to regard it as altogether satisfactory." But he has gone far in making clear to us the nature of *Principia Mathematica*. And so, we are grateful to Mr. Braithwaite for bringing close to us the writings and the personality of F. P. Ramsey.

B. A. BERNSTEIN

KRYLOFF ON MATHEMATICAL PHYSICS

Les Méthodes de Solution Approchée des Problèmes de la Physique Mathématique. (Mémorial des Sciences Mathématiques, Fascicule XLIX.) By Nicolas Kryloff. Paris, Gauthier-Villars, 1931. 69 pp.

During the present century two important methods have been developed for dealing with the boundary value problems of mathematical physics: the method of integral equations and the approximation method of W. Ritz. Integral equations are valuable for showing the existence of a solution and determining its analytical properties, but they are not well adapted to the solution of concrete problems where numerical results are wanted. The Ritz method furnishes a direct means of finding a numerical solution that is known or assumed to exist, and is especially adapted to the solution of concrete problems.

In applying the Ritz method to a physical problem, the problem is first set up as a definite integral such that the desired solution makes this integral a minimum (or maximum), just as in the calculus of variations. In the case of a function of a single variable, $y(x)$, the integrand will usually contain y and y'. The Ritz method replaces these by y_m and y_m', where y_m is a linear combination of simple functions, of the form

$$y_m = \sum_{i=1}^{m} a_i \phi_i(x), \quad (m = 1, 2, \cdots, n, \cdots, \infty).$$

Here the $\phi_i(x)$ are easily calculated functions which satisfy the boundary conditions of the problem, as $\sin m\pi x$, $x^m(1-x^2)$, etc. The coefficients a_i are determined from the condition that the integral is to be a minimum. This condition thus gives m equations which must be solved simultaneously for the a's.

Due to the labor of solving a large number of simultaneous equations, it is practically necessary to use only a few terms (not more than 6) of the series assumed for y_m. This circumstance makes it desirable that some means exist for estimating the error committed in stopping at a given number of terms. Although Ritz applied his method to a variety of problems and even solved some that had previously defied solution by all other methods, he left no means of estimating the accuracy of his results.

Since the untimely death of its brilliant author in 1909, the Ritz method has been greatly extended in usefulness and applicability by the researches of Dr. Nicolas Kryloff, and more recently with the collaboration of his pupil and assistant, Dr. N. Bogoliouboff. Kryloff has worked out explicit expressions for the upper limit of the error in the approximate solution (by the Ritz method) of several types of differential equations occurring in mathematical physics, both ordinary and partial.

The book under review gives in rather condensed form the results of Kryloff's researches on the approximate solution of ordinary differential and difference equations of the types most frequently arising in mechanics and physics. The object of the book is "to contribute to the development . . . of methods that make it possible to judge what approximation is to be taken in

order that the error committed in that approximation shall be less than a preassigned quantity." The book consists of an introduction of six pages, a first chapter of 41 pages, a second chapter of 15 pages, and an extensive bibliography.

The first chapter deals with the application of the Ritz method to the approximate solution of second-order equations containing a parameter and equations similar to the Euler equation arising in the calculus of variations. The equation

$$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + q(x)y = f(x),$$

with

$$y(0) = y(1) = 0, \quad p(x) > 0, \quad q(x) \leqq 0,$$

is first treated. It is shown that the Ritz minimizing suite y_m converges uniformly to the true solution and that the first and second derivatives of y_m converge to the corresponding derivatives of y. The convergence is proved by the method employed by Ritz himself and also by a new method which gives the limit of error at the mth approximation. This latter method gives the error $|y-y_m|$ in terms of $|y-Y_m|$, where Y_m is the sum of the first m terms of the Fourier expansion of y. Explicit expressions are given for the upper limit of $|y-Y_m|$. Indicentally a formula is given for the difference between the Ritz coefficients and the corresponding Fourier coefficients.

The next equation treated is the homogeneous equation containing a parameter,

$$\frac{d}{dx}\left[p\frac{dy}{dx}\right] + \lambda qy + ry = 0, \quad q > 0,$$

with $y(0) = y(1) = 0$. In order to estimate the error in the approximate solutions of this equation the author first derives expressions for $|\lambda_k^{(m)} - \lambda_k|$ and $|(\lambda_k^{(m)} - \lambda_k)/\lambda_k|$, where $\lambda_k^{(m)}$ denotes any one of the m roots of the secular equation obtained by equating to zero the determinant of the coefficients of the a's, and λ_k is the corresponding exact value found from a simple expression. These expressions for the error in the parameter enable one to determine in advance just how many terms of the minimizing function y_m to use in order to obtain a given degree of accuracy in the solution. Formulas are next given for computing the possible errors in the functions $y_k^{(m)}$ corresponding to the different values of λ.

The non-homogeneous equation

$$y'' + \lambda qy = f,$$

with $y(0) = y(1) = 0$, is treated in similar fashion and formulas are derived for the possible error in the Ritz solution. The chapter ends with the approximate solution of certain non-linear differential equations.

The second chapter, bearing the title "The Method of Finite Differences," is concerned with the solution of certain difference equations analogous to the differential equations previously treated, and with the same boundary conditions. The methods employed are somewhat similar to those used in the first chapter, except that interpolation formulas are used instead of Ritz functions.

The chapter closes with an article on the approximate solution of differential equations by a method of dividing the interval of integration into m equal parts and considering the coefficients constant in each subinterval. Here, as in all other cases, formulas are given for the possible error in the solution.

Throughout the book the important formulas are numbered with Roman numerals and prominently displayed. The author has supplied a correction to formula (X) as given at the bottom of page 41. The fraction $(m+1)\pi/(\lambda_\nu M)$ should be inverted.

The reviewer thinks the usefulness of the book would have been increased if the author had worked out one or two simple examples and shown the reader how to apply the more important formulas to concrete problems. Nevertheless, the book constitutes a valuable addition to the literature of analysis and mathematical physics.

J. B. SCARBOROUGH

McCONNELL ON ABSOLUTE CALCULUS

Application of the Absolute Differential Calculus. By A. J. McConnell. London and Glasgow, Blackie and Son, Ltd., 1931. xii+318 pp.

Criticism of this book had best be limited, I think, to the general impression it has made upon me and to my opinion regarding its use as a text in American colleges. In this latter connection I have had a somewhat limited experience, one of my students having studied the book as part of a course in independent reading.

The book is divided into four parts, the first part being devoted to the algebraic preliminaries including such items as the summation convention, coordinate transformations, quadratic forms, tensors, the quotient law of tensors, etc. Part II gives a treatment of the straight line, plane, and cone on the basis of rectangular coordinates; this part also contains a study of affine transformations, including strains and infinitesimal deformations. In Part III we have the differential geometry of curves and surfaces. Part IV contains applications of elementary tensor theory to the dynamics of a particle and rigid bodies, electricity and magnetism, mechanics of continuous media, and ends with a brief account of the special theory of relativity.

An objection which was immediately noticeable to me and one which I do not regard by any means as trivial is McConnell's definition of the tensor (see pp. 2, 20 and 32). McConnell has, in fact, defined the components of a tensor as the tensor itself—the tensor being the object obtained by abstraction from its components with respect to the totality of coordinate systems whose coordinates are related by the underlying family of transformations. Most of us who have written on the subject of tensor analysis have been guilty of using the symbol of the component of the tensor to represent the tensor itself; it is frequently convenient to do this and on one occasion I stated in my writings that this would be done to avoid the multiplicity of symbols which would

otherwise arise. Extreme care, however, should always be taken to avoid confusing statements in textbooks intended for elementary students; and a definition of the tensor in which the tensor is confused with its components is hardly excusable in a text on tensor analysis.

Now it seems to me that the primary purpose of an elementary book on tensor analysis should be to give the student an appreciation of the methods and point of view of this subject and that beyond that nothing should be attempted. This should be accomplished by giving the applications of the tensor analysis to geometry and mechanics as McConnell has in fact done; but McConnell has given a mass of detail in connection with this treatment of these subjects which I believe is extremely detrimental to the above mentioned purpose. For example, he has given on page 53 three forms of the equation of the plane, whereas the simplest of these, namely equation (2), would seem to be sufficient. He has undertaken a detailed study of the cases under which two planes are parallel and not parallel; he has then undertaken a similar and even more involved discussion of the case of three planes. This tendency for detail which is characteristic of the book from cover to cover contributes nothing to an essential understanding and appreciation of the applications of the tensor analysis. It is a detriment to the progress of the conscientious student who on his own responsibility attempts to read the book from beginning to end. All students, whether conscientious or otherwise, may expect difficulty in separating the essential from the unessential elements.

In the application of the tensor analysis to geometry it is, in my opinion, extremely desirable to emphasize what may be called the invariant theoretic viewpoint, that is the point of view in accordance with which we regard the study of geometry as the study of those configurations and their associated magnitude which remain invariant under the underlying group of coordinate transformations. On the basis of this point of view we can construct our invariant entities first, by the methods of the tensor analysis, and then inquire later as to their geometrical interpretation. The invariant theoretic viewpoint which is interesting and illuminating in euclidean geometry and which has proved fruitful in recent work on generalized geometries, passes unnoticed by McConnell.

With an instructor of some experience in the subject of tensor analysis, who can in fact make the proper selections and place the emphasis on the proper points, the book should not prove unsatisfactory for class room use. Under other conditions I believed that the book will be unsatisfactory for the above mentioned reasons. The binding and printing are excellent.

T. Y. THOMAS

FIVE THESES ON CALCULUS OF VARIATIONS

Contributions to the Calculus of Variations, 1930. Theses submitted to the Department of Mathematics of the University of Chicago, University of Chicago Press, July, 1931.

This volume of doctoral dissertations is of interest in the first place for its mathematical content, and secondly because it indicates a possible way of meeting the publication difficulties for theses. The University of Chicago now permits "candidates to deposit either lithoprinted or printed copies of their theses in the University library." "Owing to the moderate cost of lithoprinting extra copies it seemed an opportune time to try the experiment of distributing in collected form a limited edition of those which have been submitted during the same year or a period of years, and which are concerned with the same domain of mathematics." And so we have before us a volume of 350 pages of lithographed material, containing five theses dealing with distinct topics in the calculus of variations.

The five parts of this book treat the following topics:

1. *An envelope theorem and necessary conditions for a problem of Mayer with variable end points*, by M. G. Boyce, (pages 1–44).

2. *An historical and critical study of the fundamental lemma of the calculus of variations*, by Aline Huke, (pages 45–160).

3. *A new theory of parametric problems in the calculus of variations*, by F. L. Wren, (pages 161–194).

4. *Semi-continuity in the calculus of variations and absolute minima for isoperimetric problems*, by E. J. Mc Shane, (pages 195–244).

5. *The development of sufficient conditions in the calculus of variations*, by W. L. Duren, Jr., (pages 245–350).

In Part 1, the geometric formulation of Jacobi's necessary condition is extended to the general Mayer problem in n-space with one variable end point. The first five sections develop the multiplier-rule, and the analogues of the conditions of Weierstrass and Legendre. The general procedure is that introduced and developed by Bliss and his students; the elegance and fruitfulness of his methods become once more apparent in this work. Sections 6, 7, and 8, which contain the author's contributions to the theory, bring respectively an extension to the problem under consideration of Kneser's theorem on the envelope of a family of extremals, a geometric form of Jacobi's condition based on this theorem, and a discussion of the possibility of determining a one-parameter family of extremals which possesses an envelope with the properties needed for the application of the envelope theorem. But the difficulty which has limited the usefulness of the geometric theory in other problems again arises here; whereas it is possible to state in terms of the data of the problem conditions which insure the existence of a family of extremals with an envelope, there remains the possibility that this envelope has a singularity which interferes

with its applicability. The author limits himself, since he is concerned only with necessary conditions, to a consideration of the case in which a focal point of the hypersurface on which the variable end point is movable lies between the extremities of the extremal arc E_{12}. For the study of sufficient conditions, a further consideration of the significance of the focal point would be desirable. This paper is an interesting addition to that part of the calculus of variations which deals with Jacobi's condition, and to the study of the general Mayer problem. This study has recently been materially advanced in papers by Morse (American Journal, vol. 53 (1931), p. 517) and Bliss (Annals of Mathematics, vol. 33 (1932), p. 261). It is rather a pity that the 1927 Chicago dissertation by T. F. Cope, which deals with the same problem by an entirely different method and which has not as yet been published, was not included in the present volume.

The fundamental lemma of the calculus of variations guards, like a watchdog, the entrance gates to the entire classical domain. It has to be reckoned with by any one who wishes to advance in the subject, for it is by means of the fundamental lemma, or of a modification of the fundamental lemma, that the first-order condition of a calculus of variations problem is put in the form of a differential equation. The importance of this lemma fully justifies the attention given it by successive writers, and this in turn makes desirable a systematic survey such as forms the subject of Part 2 of the present volume.

The first two chapters, covering 17 pages, give a very interesting account of the earliest work on the fundamental lemma. It furnishes a good illustration of the variability of standards of rigor in mathematical reasoning, discussed by Professor Pierpont in this Bulletin (vol. 34 (1928), p. 23). Of particular value is the recognition of the importance of the 1848 prize memoir of Sarrus. It would take some one better versed in the history of mathematics than the present reviewer to estimate the value of these chapters critically. Chapter 3 discusses modern proofs of the lemma. The rest of this part consists of a chapter on du Bois-Reymond's lemma and its extensions for the plane (24 pages), and of a final chapter on the lemma and its extensions for multiple integrals (40 pages). This is followed by a bibliography containing 131 titles. It seems to the reviewer that in an historical account of the work of other writers, absolute accuracy of statement is a prime essential. Deviation from this standard, either due to typographical errors or to a desire to abbreviate and summarize, is to be regretted. Attention is called in this connection to the statement of du Bois-Reymond's form of δy on page 71, and to the statement of Haar's theorem on page 107. The criticism on page 99 of Kryloff's proof leads me to wonder whether the author had noticed the paper by Kryloff, not listed in the bibliography, which appeared in volume 1 of the Bulletin de l'Académie de l'Ukraïne (Classe des Sciences physiques et mathématiques, p. 8), for which a list of errata has been published. The last two chapters of this part seem to the reviewer to contain a good deal of material of less value and not to reach the level of excellence of the earlier chapters. The extensions to more general integrals of Mason's lemma (Mason's paper in Mathematische Annalen, vol. 61, closes with the sentence "Der obige Beweis lässt sich ohne weiteres auf mehrere Dimensionen ausdehnen"), of Kubota's method, and of Schauder's proof of Haar's theorem are trivial generalizations. Explicit repetition on page 129 of a page-long hypothesis

stated in full on page 124 is unfortunate, particularly since the only part of it which did require restatement was inadvertently omitted. It is not due to any fault of the author that the discussion of the proofs of Haar's theorem has lost a good deal of significance. In a paper published in the Hamburger Abhandlungen (vol. 8 (1930)), too late to be considered in this survey, Haar has given an exceedingly simple proof, together with a direct and valuable extension to the case of multiple integrals.

Part 3 develops the Hamilton-Jacobi theory for the integral $\int_{t_1}^{t_2}\sqrt{2f(x, x')}dt$, in which x and x' denote the sets $(x^1, \cdots, x^n)$ and $(x^{1\prime}, \cdots, x^{n\prime})$ respectively, the $'$ denoting differentiation with respect to t; the Euler equations for this problem were deduced by J. H. Taylor (this Bulletin, vol. 31 (1925), p. 257). The methods used in the present part follow in the main the procedure indicated by Bliss in his paper in vol. 17 (1916), of the Transactions, page 195, except that the author uses "orthogonal variations" η^α which satisfy the condition $f_\alpha\eta^\alpha=0$, where $f_\alpha=\partial f/\partial x^\alpha$, in place of the "normal variations" of Bliss. By this device a considerable simplification of the calculations is effected. The new material of this part is contained in the last 8 pages in which the classical Hamilton-Jacobi theory is carried through for a calculus of variations problem in this form. The author has not always used his own work to the best advantage; by means of the formula following (4.5) on page 176, Theorem VIII is proved in one line.

In the opinion of the reviewer Part 4 is the most significant of the volume. It gives most evidence of independent work and makes an important contribution to its field. It is devoted to the question as to the existence of curves (continuous and rectifiable) which furnish an absolute minimum for the integral $\int F(x, x')dt$ and give a prescribed value to the integral $\int G(x, x')dt$, in which x and x' represent respectively the set of functions $x_1(t), \cdots, x_n(t)$ and the set of their derivatives. The study is made by means of the "direct method," or "functional calculus" method, brought into the foreground in recent years through the work of Tonelli. The principal result is contained in Theorem V which gives a general set of sufficient conditions insuring the existence of an absolute minimum. As corollaries of this theorem are obtained extensions to n-space of the theorems in Tonelli's *Fondamenti* (vol. II, pp. 466, 468, 473, and 479); moreover an example is exhibited to which none of these theorems is applicable but for which the existence of an absolute minimum does follow from Theorem V.

There are several further results in this paper, among them generalizations to n-space of the theorems of Osgood and Lindeberg. The most important new tool introduced into the theory is the combination of Lemmas 1 (p. 207) and 2 (p. 208), according to which, whenever the sequence of continuous rectifiable curves $\{C_n\}$ with the limit curve C_0 is of uniformly bounded length, a subsequence $\{\bar{C}_n\}$ of $\{C_n\}$ can be so selected that, by proper choice of the parameter t, the sequences $F(\bar{C}_n)-F(C_0)$ and $\int_a^b E(x_0, x_0', \bar{x}_n')dt$ have the same superior and inferior limits, as $n\to\infty$; here $\bar{C}_n$ and C_0 are represented respectively by $x=\bar{x}_n(t)$ and $x=x_0(t)$, $a\leqq t\leqq b$, and, whenever $x_0'(t)=0$, it is to be replaced by an arbitrary unit vector such that x_0' remains measurable. In a later paper (Annals of Mathematics, (2), vol. 32 (1931), p. 580), the author has developed an analogous lemma for multiple integrals. There are some details con-

cerning which questions might well be raised. On page 218, line 1, there is introduced a curve Σ_n obtained "by traversing and retracing portions of $\sigma(l/n)$ enough times to make $\int_{\Sigma_n} E_\sigma(x, x', -x')dt = \int_{\Sigma_1} E_\sigma(x, x', -x')dt$." Does such a curve Σ_n always exist? In the discussion on pages 222 and 223, it would facilitate the reading considerably if it had been made clear why $h_0 \geqq H(C_0)$.

Part 5 is an historical account of the development of sufficient conditions for various problems of the calculus of variations. The survey is less critical in character than the one which constitutes Part 2; it consists more largely of a restatement of results obtained by different authors, and hence it seems to the reviewer less significant. Nevertheless it is of some interest to have the important problem of sufficiency traced systematically through the history of the subject. There are a few instances in which the reviewer would have liked to find greater accuracy of statement in the report of earlier work. The lemma attributed to Levi on page 276 is not an exact statement of the one used by that author; the multiplier rule, for which reference is given to Bliss, 1925 Chicago lectures (since then printed in American Journal, vol. 52 (1930), pp. 674–743) is put in a less general form than that given it by Bliss. It is not made clear what the exact relation is between the normality hypothesis of Bolza and that of Bliss in the sufficiency theorem stated on page 310. While the restatement in toto of the theorem of Bliss on page 308 is perhaps justifiable on the ground that the 1925 Chicago lectures were not generally available at the time this paper was written, no justification seems to exist for reproducing in full on pages 316–318 the theorem of Lindeberg of which a complete account is to be found on pages 515–519 of Bolza's *Vorlesungen*.

The various sufficiency proofs for a particular type of problem are classified according to the method of proof. After an introductory Chapter (pp. 1–14), come Chapter II on expansion methods (pp. 15–27), Chapter III on Weierstrass' field method for simple problems (pp. 28–38), and Chapter IV on methods related to the field method (pp. 39–46). The five remaining chapters deal with "more general problems," such as the problems of Lagrange and Mayer (pp. 47–63), isoperimetric problems (pp. 64–68), problems with variable end points (pp. 69–77), "problems of special types," broken extremals, one-sided variations and closed extremals (pp. 78–84), and multiple integrals (pp. 85–90). A bibliography of 121 titles closes this part. The rapid progress made the last few years in the study of sufficient conditions has made this survey incomplete even so short a time after its publication. The papers of Morse, American Journal, vol. 53, and of Bliss, Annals, vol. 33, referred to above, have served to fill the gap mentioned at the bottom of page 298. They were published after this book had been completed. On the other hand Part 4 of the book contains results which might well have been taken into account in Part 5.

This brings us to a consideration of the volume as a whole. It is perhaps a result of the circumstances which gave rise to its publication that there is no attempt to connect its different parts. Whatever unity it possesses comes from the fact that the five different parts are all concerned with the calculus of variations. Would not a good deal be gained if, in future volumes of this sort, an additional chapter were included in which the relations between the different parts, and perhaps supplementary material as well, could be considered? Some

editorial unification of the volume would also be desirable. Apart from the statement that the theses in this volume "were written under the direction of Professors G. A. Bliss and L. M. Graves", there is no indication of editorial supervision. Neither the title-page nor the unsigned preface suggest that any one has assumed responsibility for anything except his own part of the book. As might be expected under such circumstances, the proof-reading has been unevenly done. While the straight text is lithoprinted from typewritten material, all formulas and symbols not usually found on a typewriter-keyboard have been done from material written in by hand. This material shows great unevenness in legibility and in completeness. In a good many places symbols which should have been filled in have been omitted. It is true that the reviewer did not notice any misleading misprints or omissions.

It would seem that the principal gain to be derived from publishing a number of such theses jointly would lie in the attainment of a certain unity among them and in securing for them some editorial service. Should this not be feasible, the publication of the thesis in lithoprinted form could better be undertaken by each candidate individually.

A final question is in the reviewer's mind. Should doctors' theses be subjected to such critique outside the bosom of the Alma Mater which nurtured their authors as it is desirable to exercise on the publications of authors who stand entirely on their own responsibility? This is primarily a question to be considered by the Editors of the Bulletin. Had they answered it by *No*, the reviewer would have been saved a good deal of labor. It is partly because of the belief that they would answer the question affirmatively that the present review was written.

ARNOLD DRESDEN

SHORTER NOTICES

Einführung in die Analytische Geometrie und Algebra. Volume I. By O. Schreier and E. Sperner. Leipzig, B. G. Teubner, 1931. 238 pp.

This book represents the course of lectures given for several years by the talented young mathematician, O. Schreier, whose premature death was an irreparable loss to mathematics. As is seen from the title and from the brief list of contents given below, the main feature of the book is a complete fusion of the foundations of analytic geometry and algebra. This basic point of view enables the author to go more deeply into the fundamentals of analytic geometry, and to present essential material of algebra more clearly and elegantly than is usually done. Chapter I (Affine space. Linear equations) contains an exposition of general properties of vector spaces (of finite number of dimensions), the fundamentals of the theory of linear equations, and their geometric applications. Chapter II (Euclidean space. Theory of determinants) contains an exposition of most important properties of determinants and applications to the theory of transformations in geometry (rigid body motions, affine transformations, orthogonalization of systems of vectors, etc.). The last chapter (Theory of fields. Fundamental theorem of algebra) introduces the general notion of a field, and that of polynomials in a field, and gives a proof of the fundamental theorem of algebra. Since the book is supplied with numerous exercises, and the exposition is clear and elegant, it might be successfully used as a text book in this country. We await with interest the appearance of the second volume, promised within one year, which will contain the theory of matrices and linear transformations (elementary divisors), elements of the theory of groups, and projective geometry.

J. D. TAMARKIN

Leçons sur les Fonctions Entières ou Méromorphes. By Paul Montel. Paris, Gauthier-Villars, 1932. xiv+116 pp.

This book reproduces a course of lectures delivered by the author at the University of Cluj in May, 1929. The first chapter is devoted to the formula of Nevanlinna-Ostrowsky and to its applications. Nevanlinna's characteristic function is introduced and the "First fundamental theorem of Nevanlinna" is proved. In Chapter II meromorphic functions are classified according to the growth of the characteristic function. An application of the theory of the characteristic function to the canonical representation of meromorphic and entire functions is given in Chapter III. Chapter IV contains a derivation of the "Second fundamental theorem of Nevanlinna" and its applications to theorems of Picard and Borel. A rapid exposition of properties of normal families is given in Chapter V; their applications to theorems of Picard, Landau, Schottky, Julia are found in Chapter VI. The exposition follows closely the classical monograph of Nevanlinna on meromorphic functions. The reader may feel a little disappointed to find that several more recent results of great importance, announced in the Introduction, are stated without proof.

J. D. TAMARKIN

Theory of Functionals and of Integral and Integro-differential Equations. By Vito Volterra. Edited by Luigi Fantappiè. Authorized Translation by Miss M. Long. London and Glasgow, Blackie & Son, Ltd., 1930.

The book at hand is at once a translation into English and a revision and extension of a series of six lectures delivered by Volterra at the University of Madrid in 1925. Its expressed purpose is to develop in the reader an interest in and some familiarity with the theory of functionals. In this the author will in all likelihood be found highly successful.

By way of definition, z is a functional of the function $x(t)$, if under a specifiable law there corresponds one and only one value of z to each and every function $x(t)$ defined on a given interval $a \leqq t \leqq b$. The relation

$$z = \int_a^b k(t)x(t)dt,$$

in which the function $k(t)$ is specified, may be regarded as a simple example.

Beginning with definitions and generalities, the author develops his theme rapidly and carries the discussion over the problems of the functional calculus, the theory of composition and of permutable functions, functionals as a generalization of analytic functions, the theories of integral and integro-differential equations, and a survey of the extensive field of applications.

The presentation retains much of its original character as a series of lectures and is, therefore, a survey, not a treatise. The amount of detail is cut to a minimum, and is adjusted (a mark of notable expository skill) to suffice in each case for the clear presentation of the problem at issue and the method of its resolution, without, however, obscuring the general outlines on which the reasoning is planned.

For the reader versed in the general processes of analysis this book is to be recommended without reservation. An extensive bibliography covering publications to 1929 is included at the end of each lecture.

R. E. LANGER

Algebra. By Oskar Perron. Second revised edition. Vol. I, *Die Grundlagen.* Berlin and Leipzig, Walter de Gruyter, 1932. vi+301 pp. 12.80 Mk.

As the first edition of this capable general book on the fundamentals of algebra was reviewed in this Bulletin (vol. 34, p. 115), we shall here speak only of the changes that have been made in this new edition—and of changes that the reviewer wishes had been made. Taken as a whole, the new one is about the same as the old. If anyone familiar with the first edition looks at the revision, he certainly gets the impression that the latter is scarcely more than a reprint with a few secondary editorial changes.

In the third chapter, the author corrects the definition of the product of two determinants of order n from that in which we use rows of the first with rows of the second to that in which we use rows of the first with columns of the second. He still, however, makes the mistake of defining the product of two matrices (both with k rows and l columns) by using rows of the first with rows of the second, although he saves the situation by never applying this definition to

the product of three or more (where it is well known that the associative law does not hold) and by so formulating the proofs of theorems that he needs the notion of the product of two matrices precisely in this form. It is, however, regrettable that he does not give the orthodox definition of the product of two matrices (rows of the first with columns of the second), since this is the only one of the four natural possibilities (row or column of first with row or column of second) that admits the associative law and hence is the one that has to be used in applications of matrix-theory to linear transformations, group-matrices and the like. Moreover, it seems very peculiar in this day and age to omit the notion of a matrix as a hypercomplex quantity in any general book on algebra.

To this same chapter he adds several pages on the elements of linear homogeneous transformations, and these are good as far as they go, but they do not go far enough.

In his fine fifth chapter on resultants of two or more polynomials in any number of variables, he makes a few minor changes in his rather long series of theorems (overwhelming to the average beginning graduate student); but, although this is, up to the present, the best treatment of the subject by elementary methods (that is, not using Kronecker's theory of modular systems), yet one can not suppress a wish that some one would evolve a neat, adequate treatment that does not involve such a multitude of details. Moreover, use of both editions as reference books in a general graduate course on algebra makes one a little irritated at the author's apparent inability to see that Theorem 117 (§43), Theorem 118 (§44), Part II of §34, and Theorem 134 (§52) can be proved easily as corollaries of the fact (Theorem 115) that the resultant of two polynomials f and g in one variable is a polynomial in the coefficients of f and g whose vanishing is a necessary and sufficient condition that f and g have a common factor and the fact that the discriminant of a polynomial f in one variable is a polynomial in the coefficients of f whose vanishing is a necessary and sufficient condition that f have a repeated factor. Similarly, for Theorem 124 (=old Theorem 126) he uses the long-winded proof of the first edition instead of noting that this theorem follows immediately from the last equation before the statement of the theorem.

The proof of the fundamental theorem of algebra has been changed in a vital way, for instead of giving a variation of Walecki's proof, he gives the proof due to Artin, Schreier, and Dörge (Hamburg Seminar, 1927, and Sitzungsberichte der Berliner Akademie, 1928). In the application of resultants to conditions for solvability of algebraic equations in any number of variables the author adds two significant theorems (135 and 136) and the treatment of a system of resultants of a system of l non-constant polynomials in $k+l$ variables. He also adds a section (56) on the application of resultants to questions as to the factorability of a polynomial, and a section on properties of the resultant and the multiplicity of a solution.

In other words, this book remains a fine text-book in the old-fashioned highly respectable algebra, although the reviewer could not restrain a feeling of regret that the author gives no hint of the fascinating newer point of view and theory that have come into such prominence during the last five years.

OLIVE C. HAZLETT

Outline of the History of Mathematics. By Raymond Clare Archibald. Brooklyn, The Society for the Promotion of Engineering Education, 1932. 53 pp.

This *multum in parvo* of mathematical history forms No. 18 in the Selected Papers of the Summer School for Engineering Teachers issued by the Society for the Promotion of Engineering Education. It is reprinted from the Journal of Engineering Education (New Series, vol. 22, pp. 246–298, Dec., 1931) and consists of two lectures delivered before the Mathematical Session of the Summer School for Engineering Teachers at the University of Minnesota, August 31 and September 1, 1931.

The first lecture, History of Mathematics before the Seventeenth Century, covers a period of about 5000 years and the second, History of Mathematics after 1600, a period of about 300 years. The author accurately characterizes the work (p. 1) as an attempt "to give indications of the development of mathematics before the nineteenth century, and to refer briefly to some developments of the nineteenth century in connection with topics usually discussed at undergraduate colleges. Chinese and Japanese mathematics are not considered in such a skeleton survey, and the reference to mathematics of the Hindus is brief."

Since summaries and outlines are, of necessity, quite general in their nature and usually prepared to meet the needs of beginners in the field, it might be supposed that good ones are easily written. The opposite is the case. The preparation of a scholarly outline requires a large range of information, good judgment in selection of material, patience in checking carefully many details, and familiarity with the results of recent research. Fortunately, the author of this outline measures up to these qualifications. The reader may rest assured that he will find material well selected and carefully documented and that he will not find repetitions of errors which have, all too often, persisted in general secondary works long after research workers had published corrections.

U. G. MITCHELL

Vorlesungen über einige Klassen nichtlinearer Integralgleichungen und Integro-differentialgleichungen nebst Anwendungen. By Leon Lichtenstein. Berlin, Julius Springer, 1931. x+164 pp.

This monograph contains extremely rich material and introduces the reader into many theories of high importance. It represents a modified and magnified reproduction of a course of lectures delivered in 1931 by invitation at the University of Lwów. The first chapter (Nichtlineare Integralgleichungen im kleinen) contains an existence and uniqueness proof for a general class of non-linear integral equations, which includes that treated in well known papers by E. Schmidt as a special case. Instead of Schmidt's power series expansions the author consistently and successfully uses an elaborated method of successive approximations. Special attention is given to the investigation of the case of ramification (Verzweigungsfall) which plays a fundamental role in the theory and in its applications. Various extensions (for example, to integro-differential equations, systems of integral and integro-differential equations, etc.) are briefly treated. Chapter 2 gives immediate applications of the preceding theory to various problems of mathematical physics and partial differential

equations. We mention particularly the elegant solution of the problem of propagation of two-dimensional surface waves of a finite amplitude, treated by entirely different methods by Levi-Civita and several other writers; also a non-linear problem of heat conduction generalizing the one treated previously by Carleman. The end of the chapter is devoted to a discussion of solutions of the elliptic partial differential equation $\Delta z = F(x, y, z, \partial z/\partial x, \partial z/\partial y)$ considered as functionals of their boundary values. Integro-differential equations of a more general type, that can not be reduced to integral equations, are discussed in Chapter 3. Here we find a discussion of a general elliptic partial differential equation of second order containing a parameter, with applications to regular problems of the calculus of variations. Numerous applications of a general inversion principle in the theory of functional equations of a certain type are given to the problems of figures of equilibrium of rotating fluids, to dynamics of completely incoherent gravitating media, and to hydrodynamics of homogeneous incompressible perfect fluids. Chaper 4, the last, contains an application of the method of W. Ritz to the proof of the existence of characteristic values of an integral equation of the form

$$\lambda\phi(s) = \sum_{n=1}^{\infty} \int \cdots \int K_n(s; s_1, s_2, \cdots, s_n)\phi(s_1) \cdots \phi(s_n)ds_1 \cdots ds_n + f(s),$$

and of some of its modifications.

Various bibliographical references given in the book will be found very valuable by the reader. It is gratifying to find emphasis on the importance for the theory in question of numerous investigations by Liapunoff, who should be considered as a real founder of the theory of non-linear integral equations, a fact which has been partially or completely overlooked by many a writer of high authority. It is surprising, however, that important investigations of N. Günther in the existence problem of equations of hydrodynamics of a perfect fluid are not mentioned, and even that any reference to Günther is absent in the monograph.

The appearance of the book is excellent, although, even at the first reading, the reviewer was able to find quite a few misprints and lapses. On one point of some importance the reviewer finds himself to be in disagreement with the author. We have no doubt that for an intelligent reading of the book, particularly of Chapters 2 and 3, the reader will need a thorough knowledge of potential theory and of the theory of linear elliptic partial differential equations, including even quite recent developments, and many other things in addition, contrary to the author's statement in the Introduction that the "Elements of potential theory and of linear elliptic partial differential equations are desirable but not necessary, since everything that is needed is thoroughly explained in detail."

J. D. TAMARKIN

NOTES

The first issue has appeared of the Schriften des Mathematischen Seminars und des Instituts für angewandte Mathematik der Universität Berlin, under the editorship of L. Bieberbach, R. von Mises, E. Schmidt, and I. Schur. Each number will consist of a single memoir.

A new journal devoted to acoustical problems, the Revue d'Acoustique, began publication in March, 1932. The editorial committee consists of J. Brillouin, L. Brillouin, R. Lucas, A. Marcelin, and A. Proca.

The American Institute of Physics announces that it will shortly begin publication of a new periodical, the Journal of Chemical Physics, under the editorship of Professor H. C. Urey, of Columbia University.

The American Mathematical Society has appointed the following as its official delegates to the International Mathematical Congress at Zurich: Edward Kasner, C. N. Moore, R. G. D. Richardson, E. B. Stouffer, and J. D. Tamarkin.

Dr. Joseph Slepian, of the Westinghouse Electric and Manufacturing Company, has received a John Scott award, for his work in connection with gases and fundamental inventions involving these discoveries.

The Duke de Broglie has been elected an honorary member of the Cambridge Philosophical Society.

Henri Buisson and Jean Cabannes have been elected correspondents of the Paris Academy of Sciences in the section of physics.

Professor Peter Debye, of the University of Leipzig, has been elected to membership in the Royal Society of Upsala and the Accademia dei Nuovi Lincei.

Professor Albert Einstein has been elected a corresponding member of the Academy of Sciences of Lisbon.

Professor G. Fubini, of the University of Turin, has been elected a resident member of the Academy of Sciences of Turin and a national member of the Reale Accademia dei Lincei.

Professor J. S. Hadamard has been elected a foreign member of the Royal Society of London.

Professor Emile Picard has been promoted to the rank of Grand Croix of the Legion of Honor.

Professor Willem de Sitter, of the University of Leyden, has received an honorary doctorate from the University of Oxford.

Professor L. Tonelli, of the University of Pisa, has been elected a member of the Italian Society of Sciences (the XL).

The honorary degree of doctor of science has been conferred by the University of Cambridge on Sir William Bragg, director of the Royal Institution.

The University of Oxford has conferred the honorary degree of doctor of science on Sir John Russell, director of the Rothamsted Experimental Station.

Professor E. T. Whittaker, of the University of Edinburgh, has been elected a corresponding member of the Accademia dei Nuovi Lincei.

Professor E. R. Hedrick has been appointed chairman of a committee on the teaching of mathematics in the colleges and universities of the North Central Association. Professors Dunham Jackson and H. E. Slaught are also members of this committee.

Professor R. A. Millikan has received an honorary doctorate of science from Harvard University.

Professor J. M. Thomas, of Duke University, has been elected chairman of the mathematics section of the North Carolina Academy of Sciences, and Professor Helen Barton, of North Carolina College, secretary of that section.

Professor Oswald Veblen, of Princeton University, has been appointed the delegate from the American Association for the Advancement of Science to the meeting of the British Association at York, August 31 to September 7, 1932.

Professor Anna Pell Wheeler, of Bryn Mawr College, received the honorary degree of doctor of science from the New Jersey College for Women.

Professor H. S. White, of Vassar College, has received an honorary doctorate from Wesleyan University.

The following have been appointed National Research Fellows in mathematics for 1932–33: E. F. Beckenbach, J. L. Doob, R. D. James, M. H. Martin, R. S. Martin, C. W. Mendel, C. B. Morrey, S. L. Robinson, A. E. Ross, P. M. Swingle, A. W. Tucker, Hassler Whitney, S. S. Wilks. This list includes renewals.

Dr. Hermann Backhaus, of the University of Greifswald, has been appointed professor of theoretical electrotechnology at the Karlsruhe Technische Hochschule.

Dr. R. Caccioppoli, of the University of Padova, has been promoted to an associate professorship of the calculus.

Professor Hugo Dingler, of the department of mathematics of the University of Munich, has been appointed professor of philosophical pedagogics at the Darmstadt Technische Hochschule.

Dr. Karl Dörge, of the University of Cologne, has been promoted to an associate professorship of mathematics.

Professor L. Fantappiè, of the University of Palermo, has been transferred to the professorship of the calculus at the University of Bologna.

Dr. B. Finzi has been appointed associate professor of rational mechanics at the University of Milan.

Professor G. A. Maggi, of the University of Milan, has retired.

Professor M. Picone, of the University of Naples, has been transferred to the professorship of higher analysis at the University of Rome.

Associate Professor Artur Rosenthal, of the University of Heidelberg, has been promoted to a professorship of mathematics.

Dr. F. Sbrana has been appointed associate professor of rational mechanics at the University of Genoa.

Dr. B. Segre has been promoted to an associate professorship of projective and descriptive geometry at the University of Rome.

Associate Professor N. Spampinato, of the University of Catania, has been promoted to a professorship of analytic geometry.

Dr. E. A. Weiss has been promoted to an associate professorship of mathematics at the University of Bonn.

Dr. P. A. M. Dirac has been elected Lucasian professor of mathematics at Cambridge University, as successor to Sir Joseph Larmor, retired.

Mr. W. L. Edge, fellow of Trinity College, Cambridge, has been appointed lecturer in the department of mathematics of the University of Edinburgh.

Dr. E. L. Ince, of the department of mathematics of the University of Edinburgh, has resigned to accept an appointment at the Imperial College of Science, London.

Dr. W. H. McCrea, of the University of Edinburgh, has been appointed reader in mathematics at Imperial College, Royal College of Science, University of London.

The following should have been included in the list of American doctorates in mathematics for 1931, in the May number of this Bulletin:

O. W. Albert, Washington (Seattle), June, minor in astronomy, *Relations between the projective and metric differential geometries of surfaces.*

R. S. Burington, Ohio State, June, *An invariantive classification of plane cubic curves under the affine group.*

L. S. Kennison, California Institute, June, minor in mathematical physics, *Linear functional equations on a composite range.*

T. C. Bumer, Ohio State University, August, *Dynamical systems wi:h hysteresis effects of the Fredholm type.*

Dr. R. W. Barnard has been promoted to an associate professorship of mathematics at the University of Chicago.

Dr. Leonard Carlitz has been appointed assistant professor of mathematics at Duke University.

Dr. C. C. Craig has been promoted to an assistant professorship at the University of Michigan.

Dr. M. R. Hestenes has been appointed research assistant at the University of Chicago.

Dr. H. C. Hicks has been appointed to an assistant professorship at the Carnegie Institute of Technology.

Associate Professor V. B. Hinsch has been promoted to a professorship of mathematics at the Missouri School of Mines and Metallurgy.

Dr. V. A. Hoersch has been promoted to the rank of associate in mathematics at the University of Illinois.

Dr. Rosa L. Jackson has been appointed professor of mathematics at Alabama College.

Associate Professor F. W. Kokomoor, of the University of Florida, has been promoted to a professorship of mathematics.

E. D. McCarthy has been appointed assistant professor of mathematics at the University of Detroit (Engineering College).

Associate Professor Norman Miller, of Queen's University, Kingston, Ontario, has been promoted to a professorship.

Dr. David Moskovitz, of Brown University, has been appointed assistant professor of mathematics at the Carnegie Institute of Technology.

Dr. F. C. Ogg has been appointed associate professor of mathematics at Bowling Green State College, Bowling Green, Ohio.

Assistant Professor E. G. Olds, of the Carnegie Institute of Technology, has been promoted to an associate professorship of mathematics.

Dr. Mina S. Rees has been promoted to an assistant professorship of mathematics at Hunter College.

Professor B. D. Roberts, of Parsons College, has been appointed professor and head of the department of mathematics at New Mexico Normal University.

Dr. L. V. Robinson has been appointed astronomer at the Harvard College Observatory.

Dr. Otto Struve, assistant director at the Yerkes Observatory, has been appointed director of the observatory and professor of astrophysics at the University of Chicago.

Dr. C. C. Wagner has been appointed to an associate professorship at Pennsylvania State College.

Dr. Norbert Wiener has been promoted to a professorship of mathematics at the Massachusetts Institute of Technology.

The following appointments to instructorships in mathematics are announced:

University of Detroit, Mr. E. H. Johnson;
Los Angeles Junior College, Dr. S. E. Urner;
University of Oregon, Mr. A. F. Moursund;
Rutgers University, Mr. C. R. Worth;
Union College, Dr. G. B. Price;
Wilson Teachers College, Washington, D. C., Dr. C. M. Huber;
Yale University, Mr. E. P. Northrop.

The following should not have been listed as newly appointed instructors at Brooklyn College in the July number of this Bulletin: Mrs. Jennie P. Kormes, who had been instructor at Hunter College for some years and should have been listed as transferred from Hunter to Brooklyn College, and Dr. J. M. Feld, whose appointment should have been listed as in the Evening Session of Brooklyn College.

René Baire, correspondent of the Paris Academy of Sciences in the section of geometry, died July 5, 1932.

Dr. A. B. Chace, chancellor of Brown University, and editor and translator of the Rhind Papyrus, died February 28, 1932.

Associate Professor James E. Donahue, of the University of Vermont, died suddenly of cerebral hemorrhage on August 13, 1932, at the age of fifty-two years.

Dr. J. C. Fields, research professor of mathematics at the University of Toronto, died August 9, 1932. He had been a member of this Society since 1891, when the Society first adopted its present name.

Dr. Harold Jacoby, retired professor of astronomy at Columbia University, died July 20, 1932, at the age of sixty-seven. Professor Jacoby was a member of this Society at its organization as the New York Mathematical Society, and was its first treasurer.

Professor Oliver Dimon Kellogg of Harvard University, died suddenly while mountain climbing on August 27, 1932, near Greenville, Maine, at the age of fifty-four years. He had been a member of this Society since 1903, and was Vice-President during the years 1927–28. A more adequate notice will appear in an early issue of this Bulletin.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

192. Dr. A. E. Ross: *On criteria for universality of ternary quadratic forms.*

Necessary and sufficient conditions are derived in terms of generic characters that an indefinite ternary quadratic form, classic or non-classic, represent all positive and all negative integers. Such a form must represent zero properly. The theorem for classic ternary quadratic forms was proved by the author (Abstract, this Bulletin, vol. 36 (1930), p. 364) by use of a result of A. Meyer. This paper proves the theorem without use of the equivalence criterion of Meyer and similarly derives the theorem for non-classic forms. Theorem 1. Let f be a properly primitive indefinite classic ternary quadratic form with reciprocal F and determinant D. The necessary and sufficient conditions that f be universal are: D is odd or double an odd integer, $\Omega=\pm 1$, and $(F/p)=(-\Omega/p)$ for every odd prime p dividing $\Delta=D$. Theorem 2. Let f_1 be a primitive indefinite non-classic ternary quadratic form. Consider an improperly primitive form $f=2f_1$ with reciprocal F. Then f_1 is universal if and only if $f=2f_1$ satisfies the following conditions: $\Omega=\pm 1$, the characters of f are $(-1)^{(F-1)/2}=(-1)^{(-\tau-1)/2}$, $(F/p)=(-\Omega/p)$ for every odd prime p dividing $\Delta=D$, and, in case 4 divides Δ, also $(-1)^{(F^2-1)/2}=1$. These theorems serve as a practical test for determining whether a given ternary quadratic form is or is not universal. (Received July 19, 1932.)

193. Dr. C. F. Luther: *Concerning primitive groups of class u.*

In this paper are proved three general theorems concerning the degree and class of multiply transitive groups. The first gives an upper limit to the degree of a substitution group of class u that contains a substitution of order two and degree $u+\epsilon$ (ϵ a positive integer), and is more than $p_1+p_2+p_3+\cdots+p_r$ times transitive, where $p_1, p_2, p_3, \cdots, p_r$ are distinct odd prime numbers and $r>1$. It is $u>n-\epsilon-(n+2\epsilon p_1p_2\cdots p_r)/((p_1-1)(p_2-1)\cdots(p_r-1))$ if ϵ/u is small. Limits are also given for 2, 3, 5, 6, 7, and more than p (a prime)-ply transitive groups. The second theorem gives an upper limit to the degree of a triply transitive group of class $u(>3)$ that contains a substitution of degree $u+\epsilon$ (ϵ a positive integer) and of order p^ϵ (p an odd prime). The third theorem gives an upper limit to the degree of a doubly transitive group of class u that contains a substitution of degree $u+\epsilon$ (ϵ a positive integer) and of prime order p (p an odd prime). (Received July 20, 1932.)

194. Dr. Rothwell Stephens: *Continuous transformations of finite spaces.*

The following problem is considered: Given a finite space P and a knowledge of the existence or denial of eight fundamental properties in P, what possible combinations of these properties can occur in the biunivocal continuous transforms of P. The eight properties considered are those studied by D. McCoy (Tôhoku Mathematical Journal, vol. 33 (1920), pp. 89–116). Solutions are given for several particular types of spaces. In addition several extraneous theorems are proved. For every finite space of type *P4* of more than one element, the derived set of any set E is given by $K(E)=E+p$. Every finite space of type *P48* is homogeneous, and for every set E, $K(E)=E$. (Received July 21, 1932.)

195. Professor H. L. Miller: *On the summability of double Fourier series.*

Those writers who have applied Cesàro's method of summing series to the study of the Fourier development of a function of two variables have considered only integral orders of summability. This investigation considers non-integral orders in Cesàro's method of summing the double Fourier series. The Fourier development of a function of two variables, $f(x, y)$, integrable (L) in the region $(-\pi \leqq x \leqq \pi,\ -\pi \leqq y \leqq \pi)$, will be summable (Ck), $k>0$, to the value of the function at any point within the region at which the function is continuous, provided the function remains finite in some cross-neighborhood associated with the point. At a point of discontinuity (x_1, y_1) which lies on a straight line or curve containing all other points of discontinuity in the neighborhood of (x_1, y_1), the development of a function satisfying the same restrictions will be summable (Ck), $k>0$, to a value half way between the limiting values of the function as the point (x_1, y_1) is approached from either side of the line of discontinuity, provided these limiting values exist, except when the tangent to the curve of discontinuity at (x_1, y_1) is parallel to an axis. In this case the series will be restrictedly summable. (Received July 22, 1932.)

196. Professor G. S. Bruton: *Certain aspects of the theory of equations for a pair of matrices.*

Two square matrices A and B with characteristic values α_i and β_i are said to have property P_3 if any polynomial, $f(A, B)$, in A and B has its characteristic values among the numbers $f(\alpha_i, \beta_i)$. Sufficient conditions that A and B have property P_3 are found in terms of common invariant direction. Necessary and sufficient conditions are found for the 1st, 2d, and 3d order cases. Special studies are made where $f(A, B)$ is limited to being $A+B$ or AB. (Received July 22, 1932.)

197. Professor M. H. Ingraham: *A study of certain related pairs of square matrices.*

Dr. G. S. Bruton studied matrices A and B with characteristic values α_i and β_i and such that $f(A, B)$ had characteristics $f(\alpha_i, \beta_i)$. Further necessary condi-

tions that two matrices A and B have this property are found and properties of such pairs are discussed. (Received July 22, 1932.)

198. Professor T. H. Hildebrandt and Dr. I. J. Schoenberg: *On linear functional operations and the moment problem for a finite interval in one or several dimensions.*

In the first part of this paper it is shown that the Riesz theorem that a linear continuous functional operation on continuous functions is expressible in the form $\int_0^1 f d\alpha$, is deducible from and consequently equivalent to the Hausdorff theorem giving necessary and sufficient conditions that a function of bounded variation on a finite interval be determined by its moments $\int_0^1 x^n d\alpha$. This suggests in turn a simple direct proof of the Riesz theorem. The second part of the paper derives results related to Stieltjes integrals in two variables and applies them to the extension of the Hausdorff-Schoenberg results on the moment problem as well as the Riesz theorem in more than one variable. (Received July 22, 1932.)

199. Professor Glenn James: *On Fermat's last theorem.*

This paper considers the Fermat equation $x^n+y^n=z^n$, $z>y>x>0$, for the so called first case and proves that $z-y>c^n n^n$ where c is a certain function of x, y and z whose lower limit is 2. This work provides a simple, and what seems to be a new proof for the case $n=3$, and suggests a point of attack on the general problem. (Received July 23, 1932.)

200. Professor W. A. Manning: *The degree and class of multiply transitive groups*, III.

Let n be the degree and u the class of a group of substitutions G that is neither alternating nor symmetric. The author has stated that if G is 5-ply transitive, $n \leqq 2u$; if 6-ply transitive, $n<5u/3$; if 8-ply transitive, $n<8u/5$; and finally, if G is as much as 12 times transitive, $n<3u/2$. The proofs of the last two limits depend on the proposition that if in a $t(>6)$ times transitive group, all the substitutions of degree $u+\epsilon$ or less are of odd order, then $n<2u-4\epsilon-2t+13$. This is used in combination with the limits found by Dr. C. F. Luther for the degree of t-ply transitive groups of class u in which there are substitutions of degree $u+\epsilon$ of even order. (Received July 23, 1932.)

201. Mr. H. M. Bacon: *An extension of a certain theorem of Kronecker.*

A certain theorem of Kronecker may be stated as follows: Constants $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n$ being linearly independent while $\mu_1, \mu_2, \mu_3, \cdots, \mu_n$ are arbitrarily given real numbers, there exists a real number t such that differences $\alpha_j t-\mu_j$, $(j=1, 2, 3, \cdots, n)$, are in absolute value and modulo 1 less than any arbitrarily preassigned number ϵ. This paper shows that if $\epsilon=1/N$ is given, and the constants $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n$ are not linearly independent, then the inequalities $|\alpha_j t-\mu_j|<\epsilon$ (mod 1) may still be satisfied by a real number t provided all relations $c_1\alpha_1+c_2\alpha_2+c_3\alpha_3+\cdots+c_n\alpha_n=0$, with $|c_j|$ less than a

certain limit depending upon ϵ, are excluded. It is necessary to exclude all such relations having all $|c_i| \leqq N/(2n)$. It is sufficient to exclude all such relations having at least one of the $|c_i| < h \cdot N$ where $h = (\sqrt{n}/2)(125/48)^{(n^2-n)/12}$. The sufficient condition follows from the application of the Minkowski theory of reduction of quadratic forms to a problem in the geometry of number. (Received July 25, 1932.)

202. Dr. D. H. Lehmer: *A number theoretic machine.*

The number theoretic machine, whose description follows, is designed to handle any problem whose solution is restricted modulo any prime p to approximately $p/2$ cases, as for example in the problem of representing a number by a binary quadratic form using Gauss' method of exclusion. Each prime $p \leqq 113$ is represented by a gear, and each of the possible remainders modulo p corresponds to a hole in the gear. These 30 gears revolve together and ultimately arrive at a position where there is an alignment of holes. This enables a beam of light to pass through all the gears and to fall on a photo-electric cell which automatically stops the machine, so that the solution may be read. The machine sifts 20,000,000 numbers an hour without attention, and runs on the average some 50 hours between solutions. This machine was constructed with funds granted by the Carnegie Institution of Washington. (Received July 25, 1932.)

203. Dr. D. H. Lehmer: *An inversive algorithm.*

Viggo Brun has given an algorithm for calculating the nth prime number from certain values of the function $\pi(x)$, the number of primes $\leqq x$. This paper shows that this algorithm is not peculiar to primes, but on the contrary may be used to exhibit the nth member of any infinite class of positive integers. With this degree of generality it is possible to use the algorithm to obtain identities between numerical functions. (Received July 25, 1932.)

204. Professor Morgan Ward: *The arithmetical theory of linear recurring series.*

This paper considers a sequence of integers defined by a linear recursive relation to any integral modulus m, and reduces the fundamental problem of determining the least period of such a sequence modulo m essentially to determining the period of a mark in a finite Galois field. In the course of the investigation several subordinate problems of arithmetical interest connected with the periods of such sequences are solved. The results are obtained without the use of ideals by transforming the problems into equivalent ones in the theory of congruences to a double modulus which admit of complete theoretical solution. (Received July 25, 1932.)

205. Professor Morgan Ward: *The cancellation law in the theory of congruences to a double modulus.*

This paper shows how to determine all polynomials $U(x)$ such that $A(x)U(x) \equiv 0$ modd m, $F(x)$, where $A(x)$ and $F(x)$ are given polynomials with integral

coefficients and m an integer prime to the leading coefficient of $F(x)$. The problem is first reduced to the case when m is a power of a prime p and $F(x)$ is congruent modulo p to a power of an irreducible polynomial modulo p. If $m=p^n$, and p^r is the first elementary divisor corresponding to the prime p of the matrix of the eliminant of $A(x)$ and $F(x)$, then when $n \geqq r$, $U(x)$ must be of the form $p^{n-r}(A_1(x)V_1(x)+pA_2(x)V_2(x)+\cdots+p^{r-1}A_r(x)V_r(x))$ where the polynomials $A_i(x)$ are known and the polynomials $V_i(x)$ are arbitrary. The result when $n \leqq r$ is similar. (Received July 25, 1932.)

206. Professor Morgan Ward: *A type of abstract arithmetic allied to a Boolean algebra.*

This paper investigates an abstract associative, commutative, and idempotent operation, of which finding the L.C.M. and G.C.D. of two numbers are instances, with a view to ascertaining under what conditions the elements of a set closed with respect to such an operation may be uniquely resolved into prime factors in the sense of elementary arithmetic. The results may best be summarized by thinking of the elements of the set as whole numbers and the operation as that of finding the L.C.M. If it is postulated that every element has only a finite number of non-equivalent divisors, the G.C.D. operation can be defined over the system in terms of the L.C.M. If in addition it is assumed that the L.C.M. operation is distributive with respect to the G.C.D., unique factorization is obtained. The system thus obtained is contrasted with a Boolean algebra and shown to form a distinct species of non-numerical algebra. (Received July 25, 1932.)

207. Dr. E. J. Purcell: *Involutorial space Cremona transformations determined by non-linear null reciprocities.*

Birational correspondences of space are treated, for the greater part analytically, in which any two corresponding points are reciprocal with respect to a quadric polarity, or in a null-system. This paper is particularly concerned with the involutorial cases. (Received July 25, 1932.)

208. Dr. A. E. Ross: *On representation of integers by ternary forms.*

This paper considers representation of integers by ternary quadratic forms. A very simple relation is established between the generic characters of such a form f and the arithmetical progressions of integers (not necessarily prime to the determinant D of f) not represented by f. The ternary forms of different genera cannot represent the same totality of integers. In the case of indefinite ternary forms, Meyer's (A. Meyer, Journal für Mathematik, vol. 108 (1891), pp. 125–39) criterion for equivalence is employed to obtain general theorems on representation. In studying ternary forms a method is introduced which reduces the problem of representation of integers by a general positive or indefinite ternary quadratic form of invariants Ω, Δ to that of representation of multiples of appropriately chosen primes p, q by a form $2^k px^2+\Omega y^2+\Omega\Delta 2^l qz^2$, where k and l each are equal to either 0 or 1. (The method has been generalized

to apply to forms in n variables.) In the case of positive forms this latter method permits a similar reduction of the problem of determination of the number of representations. For example $N(x^2+2y^2+2z^2-2yz=m)=N(x^2+2y^2+3z^2=2m)$. (Received July 25, 1932.)

209. Miss Alta Odoms: *On the summability of triple Fourier series.*

This paper applies the Cesàro method of summation, particularly for positive non-integral orders, to the Fourier development of a function of three variables, $f(x, y, z)$. The functions considered are integrable in the sense of Lebesgue and are finite in a cross region associated with the point under consideration. It is shown that the Fourier development of such a function is summable (Cr), for any value of $r>0$, at every point of continuity of the function interior to the region $(-\pi\leqq x\leqq\pi,\ -\pi\leqq y\leqq\pi,\ -\pi\leqq z\leqq\pi)$ to the value of the function at that point. It is also shown that, if (x_1, y_1, z_1) is a point of discontinuity of $f(x, y, z)$ such that every other point of discontinuity in its neighborhood lies on a plane through (x_1, y_1, z_1) or on a curved surface through (x_1, y_1, z_1) whose tangent plane at (x_1, y_1, z_1) is not parallel to any of the coordinate axes, and if $f(x, y, z)$ approaches a definite value as (x_1, y_1, z_1) is approached from either side of the surface, the Fourier development of $f(x, y, z)$ is summable (Cr), for any value of $r>0$, at (x_1, y_1, z_1) to a value half-way between the limiting values of the function at that point. Other cases of discontinuity discussed offer more complicated results. (Received July 25, 1932.)

210. Dr. Max Coral: *On extremal surfaces.*

New proofs are given for the conditions of Haar-Lagrange, Legendre, and Weierstrass which are necessary in order that a surface $z=Z(x, y)$ of class C' in a region B may minimize an integral $I=\iint_B f(x, y, z, p, q)dxdy$ in a given class of surfaces with the same boundary. The properties of the minimizing surface are studied in a rectangular neighborhood $x_1\leqq x\leqq x_2,\ y_1\leqq y\leqq y_2$, of an interior point of the surface by substituting $z=Z(x, y)+X(x)Y(y)$ in the integral I, X and Y being functions of class C' which vanish at the ends of their respective intervals, and carrying out the integration as to y. The resulting integral $J=\int_{x_1}^{x_2}g(x, X, dX/dx)dx$ has a minimum for $X(x)\equiv 0$ in the class of functions $X(x)$ of the above described type, and the well known necessary conditions for this simple variation-problem may be interpreted for the original two-dimensional problem. The method extends to variation-problems with integrals of any multiplicity and containing any number of dependent variables. (Received July 25, 1932.)

211. Professor B. A. Bernstein: *Remarks on propositions* *1.1 *and* *3.35 *of Principia Mathematica.*

The writer's aim is to show that the interpretation of proposition *3.35 in Whitehead and Russell's *Principia Mathematica* is inadmissible; to show that the *Principia's* account of the nature of *1.1 and its relation to *3.35 is inadmissible; and to make clear the relation to *1.1 of his own proposition 1.1, given by him previously (this Bulletin, vol. 37, pp. 480–488) as the Boolean form of *1.1. (Received July 25, 1932.)

212. Professor A. A. Shaw: *The place of Ananiah Shiragooni in the history of mathematics.*

This is a continuation of the author's paper read before the November meeting of 1931, where Ananiah Shiragooni was quoted as the only source of reference available. Judging from the Jerusalem MS, Shiragooni's mathematical work is divided into three headings: (a) Concerning Weights and Measures, where he gives eighteen tables, six of which give both multiples and submultiples of the measures in question. (b) Concerning Measures of Capacity, with nineteen tables. In both classes of tables Shiragooni gives equivalent names in Hebrew, Greek, Latin, Armenian, and Egyptian with occasional etymology. These tables are in general accurate. The author found only three errors, which may be scribal. (c) Arithmetical problems, forty in number, usually very long and descriptive and referring to war, hunting, fishing, commerce, church, wine, fruits, fountain problems, etc. Only the answers are given in Armenian alphabetic numerals, as throughout the tables. No hint is given regarding the method of operations in these numerals. Probably simple operations were performed in those symbols and complicated ones by use of the abacus. Comparing the Jerusalem and the Venice MSS of Shiragooni, the author finds interesting variant forms of numerals in corresponding tables. (Received July 25, 1932.)

213. Professor Raymond Garver: *Concerning the limits of a measure of skewness.*

In a note in the Annals of Mathematical Statistics (vol. 3 (1932), pp. 141–142), Hotelling and Solomons have proved that the measure of skewness (mean —median)/standard deviation must lie between -1 and 1. In this note another proof is given; its interest lies in the fact that the theorem is shown to be essentially a generalization of a well known algebraic inequality. (Received July 26, 1932.)

214. Professor Raymond Garver: *The Edgeworth taxation phenomenon.*

In a number of papers Edgeworth has considered an interesting problem in mathematical economics. A monopolist is offering two rival commodities in the same market; a tax is imposed on the more expensive of the two. Edgeworth wished to show that it might then be (though admittedly it would not ordinarily be) to the advantage of the monopolist to *lower* both prices. In this paper the demand functions used by Edgeworth are criticized; certain of their economic implications seem to render them unsuitable. Another example is then developed, in which simpler demand functions are used, but which requires more complicated expressions for cost of production than those employed by Edgeworth. (Received July 26, 1932.)

215. Professor Raymond Garver: *Remarks on continued fractions.*

(1) Limits for the accuracy of the nth convergent in the simple continued fraction expansion of the square root of an integer are found which are usually

considerably better than the limits ordinarily given. (2) Two methods for approximating square roots are analyzed by comparing them with the continued fraction method. Some results of this kind were given by Günther in 1882. (Received July 26, 1932.)

216. Dr. I. J. Schoenberg: *On finite-rowed systems of linear inequalities in infinitely many variables*, II.

In a previous paper with the same title (Transactions of this Society, July 1932) a certain class of systems of linear inequalities in infinitely many variables was solved and applications to the theory of completely monotonic functions were derived from a particular type of such systems which were called Hausdorff systems. The present paper gives an extension of these results to a similar class of systems of linear inequalities involving a double sequence of variables. As an application of these results, the theorems of F. Hausdorff, S. Bernstein, and D. V. Widder concerning completely monotonic functions of one variable are extended to such functions of two independent variables. In particular, explicit expressions of such functions in any finite or infinite rectangular region are obtained. This paper will appear in the Transactions of this Society. (Received July 26, 1932.)

217. Professor G. T. Whyburn: *Note on a homogeneity property.*

A continuum M will be said to be arc-homogeneous provided that if s and t are any two simple arcs lying in M then there exists a topological transformation of M into itself throwing s into t. In this note it is shown that every compact locally connected and arc-homogeneous continuum of dimension 1 is a simple closed curve. A similar conclusion may be obtained even under a weaker and localized arc-homogeneity condition. (Received July 26, 1932.)

218. Professor H. S. Wall: *On continued fractions in which the coefficients have limiting values.*

Let (1) $1+[b_n z/1]_1^\infty$ be a continued fraction in which $b_n \neq 0$, $\lim_{n=\infty} b_{kn+i}=L_i$, $i=1, 2, \cdots, k$. Let α_n/β_n be the nth convergent of $1+[L_n z/1]_1^\infty$ ($L_{i+mk}=L_i$), and set $Z(z)=(-1)^{k-1}z^k L_1 L_2 \cdots L_k/(\beta_k-\beta_{k-1}+\alpha_{k-1})$.[2] Then (1) converges over every bounded closed region containing no point z_0 which is a pole of Z, or such that $Z(z_0)$ is real and $\leq -1/4$, with the exception of certain isolated points which are poles of the otherwise analytic limit. For $k=1, 2$ this gives two theorems of E. B. Van Vleck. A sufficient condition proved by Van Vleck that (1) shall be a meromorphic function is (2) $\lim b_n=0$. He showed the necessity of the condition when $b_{2n}b_{2n+1}>0$, b_n real. If some but not all the L_i are 0, and $\beta_k-\beta_{k-1}+\alpha_{k-1}\equiv 1$, (1) is meromorphic yet (2) fails. Another sufficient condition is found which requires $\lim b_{2n}q_n=\lim (b_{2n+1}/q_n)=0$, q_n a certain function of $b_1, b_2, \cdots$ and a parameter α, which reduces to 1 if $\alpha=0$. (Received July 26, 1932.)

219. Professor A. D. Michal and Dr. R. S. Martin: *Invariant functionals of functional forms.*

This paper continues the investigations of Professor Michal (A. D. Michal, American Journal of Mathematics, 1928) on functional invariants of functional forms. A number of uniqueness theorems involving Stieltjes integrals are proved. The paper includes theorems on groups of transformations that leave functional forms invariant. (Received July 26, 1932.)

220. Professor A. D. Michal and Mr. J. L. Botsford: *Simultaneous differential invariants of an affine connection and a general linear connection.*

This paper deals with invariants depending on a general König linear connection and its derivatives as well as on a symmetric affine connection and its derivatives. The properties of "geodesic representations of order r" are deduced and applied to the derivation of König normal tensors and König extensions of composite tensors. These developments are used to prove a replacement theorem for the simultaneous tensor differential invariants of the aforementioned König connection and affine connection. This theorem includes as a special case the Thomas-Michal replacement theorem for differential invariants of a symmetric affine connection (Annals of Mathematics, vol. 28 (1927), pp. 196–236; A. D. Michal, this Bulletin, vol. 36 (1930), pp. 541–546). (Received July 26, 1932.)

221. Mr. A. H. Clifford: *Geometry of a normed abelian group manifold.*

Collineations and motions in a normed Abelian group manifold are considered in this paper. Several theorems on groups of collineations and motions are proved. Specializations of the above results give interesting theorems in various functional geometries. (Received July 26, 1932.)

222. Dr. W. Cauer: *The Poisson integral for functions with positive real part.*

A representation is given for a function analytic in a half-plane and having non-negative real part there. This representation is analogous to and derived from the Poisson-Stieltjes integral, well known in the case where the function is analytic in the unit circle. The paper will appear in full in an issue of this Bulletin. (Received July 28, 1932.)

223. Professor G. A. Bliss and Dr. I. J. Schoenberg: *On the derivation of necessary conditions for the problem of Bolza.*

In this paper it is shown that the methods used by Bliss to derive the first necessary conditions for the problem of Lagrange with variable end-points (American Journal of Mathematics, vol. 52, pp. 690–693) can be adapted with very slight modifications to deduce analogous theorems for the problem of Bolza (see Bliss, Annals of Mathematics, (2), vol. 33, pp. 261–274). It is further shown how, by the introduction of some new variables, the problem of Bolza in the parametric form considered by Morse and Myers (Proceedings of the American Academy of Arts and Sciences, vol. 66, pp. 235–253) can be im-

mediately reduced to one of the Bolza type. The necessary conditions indicated by Morse and Myers, as well as their criteria concerning normality, then turn out to be simple corollaries of the corresponding theorems for the problem of Bolza. For the problem of Mayer as formulated by Myers (this Bulletin, vol. 38, pp. 303–312) a similar method is applicable. (Received July 28, 1932.)

224. Professor James McGiffert: *Particular solutions of* $y''+c_1x^py'+c_2x^{p-1}y=0$, *which consist of the product of a polynomial by an exponential.*

By application of infinite series, one obtains the following solution, $y=[x^{k-p}-((1/c_1)(k-p)(k-p-1)/(p+1))x^{k-2p-1}+(1/c_1)^2\{(k-p)(k-p-1)\cdot(k-2p-1)(k-2p-2)/((p+1)(2p+2))\}x^{k-3p-2}+\cdots]e^{\mu}$, where μ is equal to $-c_1x^{p+1}/(p+1)$, and where $k=c_2/c_1$. The expression in the brackets, multiplying the exponential portion of this solution, will be a polynomial in x for all positive integral values of p, and for all values of k such that $k=I(p+1)$ or $k+1=I(p+1)$, where I is a positive integer. For all other values of k, the expression in brackets will be found to be an infinite series, and the solution can then not be of finite form. (Received July 30, 1932.)

225. Dr. Charles Wexler: *On real quadratic fields.*

It is not unusual, in computing the fundamental units of real quadratic fields by the classical method, to find that the coefficients of the unit are numbers of fifteen or more digits. It is then difficult to check the accuracy of the computation in the ordinary way, which involves the squaring of the coefficients. In this paper are proved theorems that, among other things, simplify the verification of the coefficients. (Received August 3, 1932.)

226. Professor Orrin Frink: *Jordan measure and Riemann integration.*

It is shown in this paper that the Riemann integral can be defined in terms of Jordan measure. The situation is slightly more complicated than in the case of the Lebesgue integral; for example, the set of points at which a function integrable Riemann is $>k$ is not necessarily measurable Jordan. A necessary and sufficient condition for Riemann integrability is that for all except a countable number of values of k the "Lebesgue" sets at which the function $>k$, and $<k$, are measurable Jordan. Many other similar necessary and sufficient conditions are given. As one consequence it is shown that a function integrable Riemann can be uniformly approximated by functions which take on only a finite number of values, at sets measurable Jordan. (Received August 25, 1932.)

227. Mr. Walter Leighton: *On the convergence of continued fractions in which the elements are independent variables.* Preliminary report.

Let A_n/B_n be the nth convergent of (1) $x_0+[1/x_n]_1^{\infty}$, a continued fraction in which $x_0, x_1, \cdots$ are arbitrary complex variables, and set $g_n=\sum_{i=0}^{n}x_{2i}$. Then if the two series $\sum[x_{2n}/(g_{n-1}g_n)]$ and $\sum[x_{2n-1}g_{n-1}^2]$ converge absolutely, there

exist quantities α, β, α_1, β_1, to which A_{2n}/g_n, B_{2n}/g_n, A_{2n+1}, B_{2n+1}, respectively converge for $n=\infty$; and which satisfy a relation of the form $\alpha\beta_1-\alpha_1\beta=\delta$. Here δ is a finite quantity which is zero if and only if $\lim_{n=\infty}|g|=\infty$, and in this case (1) converges (at least in the wider sense). Seven other convergence criteria of which this is an example are also obtained. (Received August 10, 1932.)

228. Dr. E. W. Miller: *Solution of the Zarankiewicz problem.*

The following question has been raised (Fundamenta Mathematicae, vol. 7 (1925), p. 381, problem 37) by C. Zarankiewicz: Is every acyclic continuous curve homeomorphic with some proper subset of itself? It is the purpose of this paper to show that the above question is to be answered in the negative. An acyclic continuous curve S is constructed which contains a set κ with the following two properties: (1) Each point of K is a fixed point with respect to any (1, 1) bicontinuous transformation of S into a subset of itself. (2) Each point of S of (Urysohn-Menger) order >1 lies on an arc of S whose end points are points of K. It is easily shown that such an acyclic continuous curve can be homeomorphic with no proper subset of itself. (Received August 29, 1932.)

229. Dr. W. M. Rust: *A theorem on Volterra integral equations of the second kind with singular kernels.*

The Volterra integral equation of the second kind with kernel $K(x, y)$ which satisfies the following two conditions: (i) if $r(x)$ is bounded and continuous so is $\int_0^x K(x, y)r(y)dy$, (ii) for n sufficiently great the iterated kernel $K_x^n(x, y)$ is bounded and continuous, has a unique, bounded, continuous solution which is the solution of the equation (a) $u_n(x)=\sum_{m=1}^{2n-1}\int_0^x K_m(x, y)f(y)dy+\int_0^x K_{2^n}(x, y)u_n(y)dy$, $f(x)$ being bounded and continuous. The proof, which is elementary, consists in showing that a bounded and continuous solution of the equation of the type (a) involving only kernels up to $K_x^i(x, y)$ is also a solution of the corresponding equation involving only kernels up to $K_x^{i-2}(x, y)$. (Received August 19, 1932.)

230. Professor Norbert Wiener and Dr. R. C. Young: *The total variation of $g(x+h)-g(x)$.*

A new proof is given of Plessner's theorem, that $f(x)$ is absolutely continuous when and only when the total variation of $f(x+h)-f(x)$ tends to 0 with h. The set of values of h for which the total variation of $f(x+h)-f(x)$ is not the sum of the total variations of $f(x)$ and $f(x+h)$ is null if $f(x)$ is constant in the complementary intervals of a closed null set. The null set of values of h in question may be denumerable, finite (including non-existent), or of the order of the continuum. Examples of all three possibilities are given, the denumerable case being due to Hille and Tamarkin. (Received August 2, 1932.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

DE ALMEIDA E VASCONCELLOS (F.). História das matemáticas na antiguidade. Lisboa, 1931.

BANACH (S.). Théorie des opérations linéaires. (Monografje Matematyczne, I.) Warsaw, Mathematical Seminar of the University of Warsaw, 1932. 8+252 pp.

BESICOVITCH (A. S.). Almost periodic functions. Cambridge, University Press, 1932. 13+180 pp.

BIEBERBACH (L.). Analytische Geometrie. 2te verbesserte und vermehrte Auflage. Leipzig, Teubner, 1932. 6+141 pp.

BOREL (E.) et DELTHEIL (R.). La géométrie et les imaginaires. Paris, Albin Michel, 1932. 310 pp.

BOSANQUET (B.). Logic; or, the morphology of knowledge. 2d edition. 2 volumes. Oxford, Clarendon Press, 1932.

BROWN (W. L.). Related mathematics. New York, Wiley, 1932.

BURKE (J. B.). The mystery of life. London, Elkin Mathews and Marrot, 1931. 160 pp.

CARROLL (L.). Catalogue of an exhibition at Columbia University to commemorate the one hundredth anniversary of the birth of Lewis Carroll (Charles Lutwidge Dodgson) 1832–1898. New York, Columbia University Press, 1932. 153 pp.

DEDEKIND (R.). Gesammelte mathematische Werke. Herausgegeben von R. Fricke, E. Noether, O. Ore. Band 3. Braunschweig, Vieweg, 1932. 508 pp.

DELTHEIL (R.). See BOREL (E.).

DURELL (C. V.). Advanced algebra. Volume 1. London, Bell, 1932. 8+193+22 pp.

FRICKE (R.). See DEDEKIND (R.).

GANGUILLET (V.). Ueber multiplikatorisch periodische Funktionen. (Dissertation, Zürich.) Zürich, 1931. 33 pp.

GÖLZ (A.). Ueber die hyperbolischen Eckpunkte bei Normalpolygonen Fuchs'scher Gruppen mit Angabe einiger Beispiele. (Dissertation, Zürich.) Zürich, 1931. 60 pp.

HEMMINGS (F. J.). The teaching of arithmetic and elementary mathematics. London and Glasgow, Blackie, 1932. 7+156 pp.

HERBENER (E.). Ueber Kongruenz- und Transformationsgruppen und definite Hermite'sche Formen im Körper $k(\sqrt{-1})$. (Dissertation, Zürich.) Zürich, 1930.

HOSTINSKY (B.). Méthodes générales du calcul des probabilités. (Mémorial des Sciences Mathématiques, No. 52.) Paris, Gauthier-Villars, 1931. 66 pp.

INTERNATIONAL MATHEMATICAL CONGRESS. Atti del Congresso Internazionale dei Matematici, Bologna, 3–10 Settembre 1928. Tomo IV e tomo V. Bologna, Zanichelli, 1932. 429+494 pp.

JONES (S. I.). Mathematical nuts for lovers of mathematics. Nashville, published by the author, 1932. 12+340 pp.

KAMKE (E.). Einführung in die Wahrscheinlichkeitstheorie. Leipzig, Hirzel, 1932. 8+182 pp.

KNOPP (K.). Aufgabensammlung zur Funktionentheorie. 2te verbesserte Auflage. Teil II. (Sammlung Göschen.) Berlin, de Gruyter, 1932. 138 pp.

LANE (E. P.). Projective differential geometry of curves and surfaces. Chicago, University of Chicago Press, 1932. 12+321 pp.

LAPLACE (P. S.). Philosophische Versuch über die Wahrscheinlichkeit. (Ostwalds Klassiker der Exakten Wissenschaften.) Leipzig, Akademische Verlagsgesellschaft, 1932. 8+212 pp.

LEHMANN (G. A.). Ueber die Koeffizienten in den Substitutionen der Zerlegungs-, Trägheits- und Verzweigungsgruppen. (Dissertation, Zürich.) Zürich, 1931. 75 pp.

LOCHER (L.). Ueber Gruppen konformer Raumabbildungen und Modulfunktionen des Raumes. (Dissertation, Zürich.) Zürich, 1930. 26 pp.

LONDON MATHEMATICAL SOCIETY. Notes on the preparation of mathematical papers. London, Hodgson, 1932. 19 pp.

MICHEL (C.). Compléments de géométrie moderne. Paris, Vuibert, 1932. 320 pp.

MOORE (R. L.). Foundations of point set theory. (American Mathematical Society Colloquium Publications, volume 13.) New York, American Mathematical Society, 1932. 8+486 pp.

MÜLLER (M.). Individualité, causalité, indéterminisme. Paris, Alcan, 1932. 204 pp.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. Fifth yearbook: The teaching of geometry. Sixth yearbook: Mathematics in modern life. Seventh yearbook: The teaching of algebra. New York, Teachers College, Columbia University, 1930, 1931, 1932. 12+206+13+126+179 pp.

NESSI (A.) et NISOLLE (L.). Calcul graphomécanique. Appareils pour le calcul mécanique de l'intégrale du produit de deux fonctions. Paris, Dunod, 1932.

NILLUS (P.). Leçons de calcul vectoriel. Paris, Eyrolles, 1931. 347 pp.

NISOLLE (L.). See NESSI (A.).

NOETHER (E.). See DEDEKIND (R.).

ORE (O.). See DEDEKIND (R.).

REIDEMEISTER (K.). Einführung in die kombinatorische Topologie. Braunschweig, Vieweg, 1932. 12+209 pp.

RITT (J. F.). Differential equations from the algebraic standpoint. (American Mathematical Society Colloquium Publications, volume 14.) New York, American Mathematical Society, 1932. 10+172 pp.

ROHRBERG (A.). Algebra. Oberstufe. München, Oldenbourg, 1931. 6+200 pp.

RUTHERFORD (D. E.). Modular invariants. (Cambridge Tracts, No. 27.) Cambridge, University Press, 1932. 8+84 pp.

SARASIN (M. L.). Ueber linear gebrochene Quaternionensubstitutionen und die Abbildungen des Hyperraumes. (Dissertation, Zürich.) Zürich, 1930. 43 pp.

SKINNER (E. B.). Introduction to trigonometry and analytic geometry. New York, Macmillan, 1932. 189 pp.

STEIN (J.). Eine neue Darstellung aller rationellen Zahlen, einschliesslich der komplexen Zahlen und der Zahl unendlich. Hamburg, Lütcke und Wulff, 1932. 16 pp.

THUENER (M. D.). On the number and reality of the self-symmetric quadrilaterals in- and circumscribed to the triangular-symmetric rational quartic. (Dissertation, Catholic University.) Washington, Catholic University of America, 1932. 19 pp.

PART II. APPLIED MATHEMATICS

AHLBERG (H.). Festigkeitslehre in elementarer Darstellung mit zahlreichen, der Praxis entnommenen Beispielen. 9te verbesserte Auflage. Leipzig, Jänecke, 1932. 153 pp.

ALBERTI (E.). Braunsche Kathodenstrahlröhren und ihre Anwendung. Berlin, Springer, 1932. 7+214 pp.

ALLCUT (E. A.) and MILLER (E.). Materials and their application to engineering design. 2d edition. London, Charles Griffin, 1932. 533 pp.

ALLEN (H. S.). Electrons and waves. An introduction to atomic physics. London, Macmillan, 1932. 8+336 pp.

ANDRADE (E.). Le mécanisme de la nature. Traduit de l'anglais par G. Malgorn. Paris, Dunod, 1932. 163 pp.

BARNECK (A.). Die Grundlagen unserer Zeitrechnung. 2te Auflage. (Mathematisch-Physikalische Bibliothek.) Leipzig, Teubner, 1932. 49 pp.

BARRACLOUGH (F.) and HOLMYARD (E. J.). Mechanics for beginners. London, Dent, 1931. 8+214 pp.

BIGGS (E.). Elementary chemical calculations. London and Glasgow, Blackie, 1932. 80 pp.

BOLL (M.). Exposé électronique des lois de l'électricité. Paris, Hermann, 1932. 72 pp.

—— L'idée générale de la mécanique ondulatoire et de ses premières applications. Paris, Hermann, 1932. 74 pp.

BRELOT (M.). Etude de l'équation de la chaleur $\Delta u = c(M)$, $c(M) \geq 0$, au voisinage d'un point singulier du coefficient. (Dissertation, Paris.) Paris, 1931. 95 pp.

BRINKMAN (H. C.). Zur Quantenmechanik der Multipolstrahlung. (Dissertation, Utrecht.) Groningen, Noordhoff, 1932.

DE BROGLIE (L.). Sur une forme plus restrictive des relations d'incertitude d'après MM. Landau et Peierls. (Exposés de Physique Théorique, I.) Paris, Hermann, 1932. 24 pp.

—— La théorie de la quantification dans la nouvelle mécanique. Paris, Hermann, 1932. 8+250 pp.

BROWN (E. W.). Elements of the theory of resonance. Illustrated by the motion of a pendulum. Cambridge, University Press, 1932. 4+60 pp.

BUCKLEY (A.) and MACDERMOTT (C. F. G.). Elementary mechanics. London, Bell, 1932. 8+216+19 pp.

CARRELLI (A.). La teoria quantistica. Roma, Scuola Tipogfrafica Pio X, 1931. 200 pp.

CORPS (—.). Ce qu'il faut penser de la théorie d'Einstein. Paris, Librairie d'Action Française, 1932.

COUDERC (P.). Dans le champ solaire. Paris, Gauthier-Villars, 1932. 7+237 pp.

CRATHORNE (A. R.). See RIETZ (H. L.).

CROWTHER (J. G.). Osiris and the atom. London, Routledge, 1932. 8+221 pp.

CURETON (E. E.) and DUNLAP (J. W.). C-D correlation chart. New York, Macmillan, 1932. 25 sheets.

CURIE (I.) et JOLIOT (F.). Le projection de noyaux atomiques par un rayonnement très pénétrant. L'existence du neutron. (Exposés de Physique Théorique, II.) Paris, Hermann, 1932. 22 pp.

DARMOIS (E.). L'hydrogène est un mélange; ortho- et parahydrogène. Paris, Hermann, 1931. 24 pp.

DEMOLON (A.). La dynamique du sol. Paris, Dunod, 1932.

DUNLAP (J. W.). See CURETON (E. E.).

DUNLAP (J. W.) and KURTZ (A. K.). Handbook of statistical nomographs, tables, and formulas. Yonkers, World Book Company, 1932. 10+164 pp.

EGGERT (J.). Physical chemistry. Translated by S. J. Gregg. A translation of the third edition of "Lehrbuch der physikalischen Chemie," revised in collaboration with L. Hock. London, Constable, 1932. 11+632 pp.

EVERSHED (W. L.). Quantity surveying for builders. A text-book for surveyors, civil engineers, builders and contractors. London, Chapman and Hall, 1932. 19+278 pp.

FISHENDEN (M.) and SAUNDERS (O. A.). The calculation of heat transmission. London, H. M. Stationery Office, 1932. 12+280 pp.

FOCH (A.). Introduction à la mécanique des fluides. Paris, Armand Colin, 1932. 200 pp.

FUSCALDO (O.). Les moteurs à deux temps rapides à explosion et à combustion. Paris, Dunod, 1932. 112 pp.

GREGG (S. J.). See EGGERT (J.).

HAUFFE (G.). Ortskurven der Starkstromtechnik. Einführung in ihre Theorie und Anwendung. Berlin, Springer, 1932. 174 pp.

HEDGES (E. S.). Liesegang rings and other periodic structures. London, Chapman and Hall, 1932. 8+122 pp.

HOCK (L.). See EGGERT (J.).

HOLMYARD (E. J.). See BARRACLOUGH (F.).

JOLIOT (F.). See CURIE (I.).

KEAL (H. M.), and others. The slide rule. New York, Wiley, 1932.

KEARTON (W. J.) and WOOD (G.). Alignment charts for engineers and students. 2d edition. London, Charles Griffin, 1932. 228 pp.

KERMODE (A. C.). An introduction to aeronautical engineering. Volume 1: Mechanics of flight. London, Gale and Polden, 1932. 223 pp.

KIESSLING (F.). Eine Methode zur approximativen Berechnung einseitig eingespannter Druckstäbe mit veränderlichen Querschnitt. (Dissertation, Uppsala.) Uppsala, Elanders Boktryckeri Aktiebolag, 1930. 64 pp.

KOHLRAUSCH (F.). Kleiner Leitfaden der praktischen Physik. 5te Auflage neubearbeitet von F. Krüger. Leipzig, Teubner, 1932. 27+498 pp.

KRÜGER (F.). See KOHLRAUSCH (F.).

KURTZ (A. K.). See DUNLAP (J. W.).

LAFITTE (J.). Réflexions sur la science des machines. Paris, Bloud et Gay, 1932. 164 pp.

LEE (D. H.). Graphs for engineers and architects. London, Spon, 1932. 96 pp.

MACDERMOTT (C. F. G.). See BUCKLEY (A.).

MALGORN (G.). See ANDRADE (E.).

MARCOLONGO (R.). La meccanica di Leonardo da Vinci. Napoli, Stabilimento Industrie Editoriali Meridionali, 1932.

MARCUS (H.). Die Theorie elastischer Gewebe und ihre Anwendungen auf die Berechnung biegsamer Platten. 2te, verbesserte Auflage. Band 1. Berlin, Springer, 1932. 8+368 pp.

MARSHALL (C. F. D.). The motion of railway vehicles on a curved line. With an appendix dealing with the curving of road vehicles. London, The Railway Engineer, 1932. 41 pp.

MICHEL (F.). Lärm und Resonanzschwingungen im Kraftwerksbetrieb infolge periodischer Strömungsvorgänge. Berlin, Verein Deutscher-Ingenieure, 1932. 84 pp.

MILLER (E.). See ALLCUT (E. A.).

MOLESWORTH (G. L.). Weights, measures and decimal tables. London, Spon, 1932. 142 pp.

MORSE (A. H.). Radio beam and broadcast. London, Benn, 1932.

VON NEUMANN (J.). Mathematische Grundlagen der Quantenmechanik. (Die Grundlagen der Mathematischen Wissenschaften, Band 38.) Berlin, Springer, 1932. 262 pp.

NOLASCO DE ALMEIDA (T.). Miguel Faraday. Rio de Janeiro, 1932. 31 pp.

OATLEY (C. W.). Wireless receivers. The principles of their design. London, Methuen, 1932. 7+103 pp.

OLLENDORFF (F.). Potentialfelder der Elektrotechnik. Berlin, Springer, 1932. 8+395 pp.

OLLIVIER (H.). Cours de physique générale. Tome 3: Mouvements vibratoires. Acoustique, Optique physique, Ondes électromagnétiques, Electro-optique, Effets optiques du mouvement. Paris, Hermann, 1932.

PAPPERITZ (E.). See ROHN (K.).

PERRIN (F.). Fluorescence. Durée élémentaire d'émission lumineuse. Paris, Hermann, 1931. 45 pp.

RAINICH (G. Y.). Mathematics of relativity. Ann Arbor, Edwards Brothers, 1932. 67 pp.

RAMSEY (C. G.) and SLEEPER (H. R.). Architectural graphic standards for architects, engineers, decorators, builders and draftsmen. New York, Wiley, 1932. 9+233 pp.

REYNER (J. H.). See WEDMORE (E. B.).

RIETZ (H. L.), CRATHORNE (A. R.), and RIETZ (J. C.). Mathematics of finance. Revised edition. New York, Holt, 1932. 16+346 pp.

RIETZ (J. C.). See RIETZ (H. L.).

ROHN (K.) und PAPPERITZ (E.). Lehrbuch der darstellenden Geometrie. 4te Auflage. 1ter und 2ter Band. Berlin, de Gruyter, 1932. 20+502+6+194 pp.

SAUNDERS (O. A.). See FISHENDEN (M.).

SCHWEDLER (F.). Handbuch der Rohrleitungen. Allgemeine Beschreibung, Berechnung, Herstellung, Normung, Tabellen und Bildtafeln. Berlin, Springer, 1932. 225 pp.

SLEEPER (H. R.). See RAMSEY (C. G.).

SNEEDEN (J. B. O.). Elements of steam power engineering. London and New York, Longmans, 1932. 8+248 pp.

SPRIGG (C.). The airship. Its design, history, operation and future. London, Sampson Low, Marston, 1932. 256 pp.

STILL (A.). Elements of electrical design. 2d edition. New York, McGraw-Hill, 1932. 603 pp.

SUDRIA (J.). Les unités électriques. Paris, Gauthier-Villars, 1932. 87 pp.

TENOT (A.). Turbines hydrauliques et régulateurs automatiques de vitesse. Livre 2. Paris, Eyrolles, 1932. 8+619 pp.

THORPE (F. J.). The Faraday books of practical science. Book 5: Light and sound. London, Oxford University Press, 1932. 110 pp.

TOCHTERMANN (W.). Das Maschinenzeichnen mit Einführung in das Konstruieren. Band 1. Berlin, de Gruyter, 1932. 154 pp.

TURNER (A. D.). Applied building mechanics. Examples of the use of calculated work in building design. Bristol, J. Wright, and London, Simpkin Marshall, 1932. 209 pp.

WALKER (M.). The control of the speed and power factor of induction motors. London, Ernest Benn, 1932.

WEDMORE (E. B.) and REYNER (J. H.). Radio engineering. London, Ernest Benn, 1932.

WEST (W.). Acoustical engineering. The theory of sound and its applications to telephone and architectural engineering and to acoustical measurements and research. London, Pitman, 1932. 11+338 pp.

WOOD (G.). See KEARTON (W. J.).

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 29, 1932.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th St., New York City, not later than October 8. In order to be printed in the BULLETIN in advance of the meeting, abstracts must be in the hands of the Associate Secretary not later than September 7. By invitation of the program committee, Professor Jesse Douglas will speak on *The problem of Plateau.*

AMES, IOWA, November 25-26, 1932.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 3. Abstracts received by that date will appear in the November issue of this Bulletin. By invitation of the program committee, Professor E. B. Stouffer will speak on *Some canonical forms and their associated geometries,* and Professor H. W. March will speak on *Applications of the theory of elasticity to wood, a material of anisotropic structure.*

ATLANTIC CITY, NEW JERSEY, ANNUAL MEETING, December 27-30, 1932.

Abstracts are due by November 30, 1932.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

WHOLE NUMBER 396

CONTENTS

PAGE

The Syracuse Meeting of Section A, American Association for the Advancement of Science. By F. F. DECKER. 601

John Wesley Young—In Memoriam. By R. D. BEETLE and C. E. WILDER. 603

Ramsey on Foundations of Mathematics. By B. A. BERNSTEIN. 611

Kryloff on Mathematical Physics. By J. B. SCARBOROUGH 613

McConnell on Absolute Calculus. By T. Y. THOMAS. 615

Five Theses on Calculus of Variations. By ARNOLD DRESDEN. 617

Shorter Notices. 622

Notes. 627

Abstracts of Papers. 632

New Publications. 643

For official Communications and Notices, see the inside of the back cover.

GEORGE BANTA PUBLISHING COMPANY, MENASHA, WISCONSIN